Bayesian Analysis of Time-Varying Parameter Vector Autoregressive Model with the Ordering of Variables for the Japanese Economy and Monetary Policy

Jouchi Nakajima (Duke University)
Toshiaki Watanabe (Hitotsubashi University)

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TVP-VAR Model

- Primiceri (2005) proposed a time-varying parameter VAR model where the both parameters and volatilities follow a random-walk process.
- He developed a Bayesian method using MCMC for the analysis of this model.
- Using this model, he analyzed the time-varying structure of the US economy.

TVP-VAR Model

Structural form

$$A_t y_t = \mu_t + F_{1t} y_{t-1} + \cdots + F_{st} y_{t-s} + \Sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0, I),$$

where

$$A_t = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ a_{21t} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ a_{k1t} & \cdots & a_{k,k-1,t} & 1 \end{pmatrix},$$

$$\Sigma_t = \begin{pmatrix} \sigma_{1t} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{kt} \end{pmatrix}.$$

TVP-VAR Model

Reduced form

$$y_t = c_t + B_{1t} y_{t-1} + \cdots + B_{st} y_{t-s} + A_t^{-1} \Sigma_t \varepsilon_t,$$

where $c_t = A_t^{-1} \mu_t$ and $B_{it} = A_t^{-1} F_{it}$.

Let

β_t: stacked vector of $(c_t, B_{1t}, \ldots, B_{st})$
a_t: stacked vector of A_t
$h_{it} = \log \sigma_{it}^2$
$h_t = (h_{1t}, \ldots, h_{kt})'$

TVP-VAR Model

We assume the random walk process for the time-varying parameters:

$$
\begin{aligned}
\beta_{t+1} &= \beta_t + u_{\beta t}, \\
a_{t+1} &= a_t + u_{at}, \\
h_{t+1} &= h_t + u_{ht},
\end{aligned}
$$

$$
\begin{pmatrix} \epsilon_t \\ u_{\beta t} \\ u_{at} \\ u_{ht} \end{pmatrix} \sim N\left(0, \begin{pmatrix} I & O & O & O \\ O & V_\beta & O & O \\ O & O & V_a & O \\ O & O & O & V_h \end{pmatrix}\right),
$$

where V_β is non-diagonal and V_a and V_h are diagonal matrices.

TVP-VAR Model

- Nakajima et al. (2011) have already applied this model to the Japanese economy.
- Then, what is new in this paper?
- Since the recursive structure is assumed for identification, the ordering of variables matters.
- We propose a method for the ordering of variables using the RJMCMC proposed by Green (1995).

RJMCMC

- Suppose that there are competing models $m = 1, \ldots, M$ and the vector of parameters in model m is denoted by θ_m.
- RJMCMC is an algorithm to sample from the joint posterior distribution $\pi(m, \theta_m | y)$ by exploiting the MH algorithm.
- We use four variables (p: inflation, y: industrial production, r: call rate, m: monetary base).
- Then, $M = 24$ as follows.

RJMCMC

No.	Order	No.	Order	No.	Order
1	(p, y, r, m)	9	(y, r, p, m)	17	(r, m, p, y)
2	(p, y, m, r)	10	(y, r, m, p)	18	(r, m, y, p)
3	(p, r, y, m)	11	(y, m, p, r)	19	(m, p, y, r)
4	(p, r, m, y)	12	(y, m, r, p)	20	(m, p, r, y)
5	(p, m, y, r)	13	(r, p, y, m)	21	(m, y, p, r)
6	(p, m, r, y)	14	(r, p, m, y)	22	(m, y, r, p)
7	(y, p, r, m)	15	(r, y, p, m)	23	(m, r, p, y)
8	(y, p, m, r)	16	(r, y, m, p)	24	(m, r, y, p)

p: inflation rate, y: industrial production, r: call rate, m: monetary base.

RJMCMC

MH algorithm

(1) Choose the proposal density $g(x)$ and the initial value x_0.

(2) Set $n = 1$.

(3) Sample $x_n^{(\text{proposal})}$ from $g(x)$. Then, calculate the acceptance probability as follows.

$$q = \min\left[\frac{f(x_n^{(\text{proposal})})g(x_{n-1})}{f(x_{n-1})g(x_n^{(\text{proposal})})}, 1\right]$$

(4) Accept $x_n^{(\text{proposal})}$ with probability q and reject it with probability $1 - q$. Set $x_n = x_n^{(\text{proposal})}$ if accepted and $x_n = x_{n-1}$ if rejected.

(5) If $n < N$, set $n = n + 1$ and goto (3). If $n = N$, end.

RJMCMC

MH algorithm

- Chib and Greenberg (1995)

RJMCMC

1. Propose a move $(m, \theta_m) \to (m^*, \theta^*_{m^*})$ as follows.
   (1) Propose m to m^* with probability $j(m, m^*)$.
   (2) Sample $\theta^*_{m^*}$ from the proposal density $q(\theta^*_{m^*}|\theta_m, m, m^*)$.
2. Accept the move $(m, \theta_m) \to (m^*, \theta^*_{m^*})$ with the MH acceptance probability $\alpha(m, m^*) = \min[1, R]$, where

$$R = \frac{f(y|m^*, \theta^*_{m^*})g(\theta^*_{m^*}|m^*)\pi(m^*)j(m^*, m)q(\theta_m|\theta^*_{m^*}, m^*, m)}{f(y|m, \theta_m)g(\theta_m|m)\pi(m)j(m, m^*)q(\theta^*_{m^*}|\theta_m, m, m^*)}$$

RJMCMC

Choice of $q(\theta^*_{m^*}|\theta_m, m, m^*)$

$$
\begin{aligned}
& q(\beta^*, a^*, h^*, V^*|\beta, h, V, y) \\
= \quad & q(\beta^*|\tilde{h}, \tilde{V}_\beta, \tilde{y}) \times q(a^*|\beta^*, \tilde{h}, \bar{V}_a, \tilde{y}) \\
& \times q(h^*|\beta^*, a^*, \tilde{V}_h, \tilde{y}) \times q(V^*|\beta^*, a^*, h^*),
\end{aligned}
$$

where $\tilde{x}$ is the permutation of x according to the order change of $m \to m^*$.

RJMCMC

$q(\beta^*|\tilde{h}, \tilde{V}_\beta, \tilde{y})$

- We assume that $A_t = A_t^{-1} = I$ for all t.
- Then, we have the linear Gaussian state-space model:

$$\begin{aligned} y_t &= X_t\beta_t + \Sigma_t\varepsilon_t, \quad \varepsilon_t \sim N(0, I), \\ \beta_{t+1} &= \beta_t + u_{\beta t}, \quad u_{\beta t} \sim N(0, V_\beta), \end{aligned}$$

  where $X_t = I_k \otimes (1, y'_{t-1}, \ldots, y'_{t-s})$.
- The state variable β is generated using the simulation smoother (de Jong and Shephard, 1995; Durbin and Koopman, 2002).

RJMCMC

$q(a^*|\beta^*, \tilde{h}, \bar{V}_a, \tilde{y})$

- We assume that $\bar{V}_a = \bar{v}_a^2 I$ with $\bar{v}_a^2 = (v_{a1}^2 + \cdots + v_{aq}^2)/q$, *i.e.*, the average of the variance in the current point of V_a.
- We have the linear Gaussian state-space model:

$$\begin{aligned} \hat{y}_t &= \hat{X}_t a_t + \Sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0, I) \\ a_{t+1} &= a_t + u_{at}, \quad u_{\beta t} \sim N\left(0, \bar{v}_a^2 I\right), \end{aligned}$$

  where $\hat{y}_t \equiv (\hat{y}_{1t}, \ldots, \hat{y}_{kt})' = y_t - X_t \beta_t$.
- The state variable a is generated using the simulation smoother.

RJMCMC

$q(h^*|\beta^*, a^*, \tilde{V}_h, \tilde{y})$

- We have the stochastic volatility model:

$$\begin{aligned} w_{it} &= \exp(h_{it}/2)\varepsilon_{it}, \quad \varepsilon_t \sim N(0, I), \\ h_{i,t+1} &= h_{it} + u_{hit}, \quad u_{hit} \sim N(0, V_{hi}), \end{aligned}$$

  where w_{it} is the i-th element of $w_t = A_t \hat{y}_t$.
- We generate h using the block sampler (Shephard and Pitt, 1997; Watanabe and Omori, 2004).

RJMCMC

Sampling h

- Mixture sampler
  - Kim et al. (1998), Omori et al. (2007)
  - This method approximates the distribution of $\log(\varepsilon_{it}^2)$ by a mixture of normals.
- Block (or multi-move) sampler
  - Shephard and Pitt (1997), Watanabe and Omori (2004)
  - This method does not require any approximation.

RJMCMC

Choice of $j(m, m^*)$

$$j(m, m^*) = \begin{cases} (1 - S)/(M - 1), & m \neq m^* \\ S, & m = m^* \end{cases},$$

where $M = 24$ and we set $S = 0.3$.

Application to the Japanese Economy

Data:

- p: price (CPI)
- y: industrial production (IIP)
- r: call rate
- m: monetary base

Sample Period:

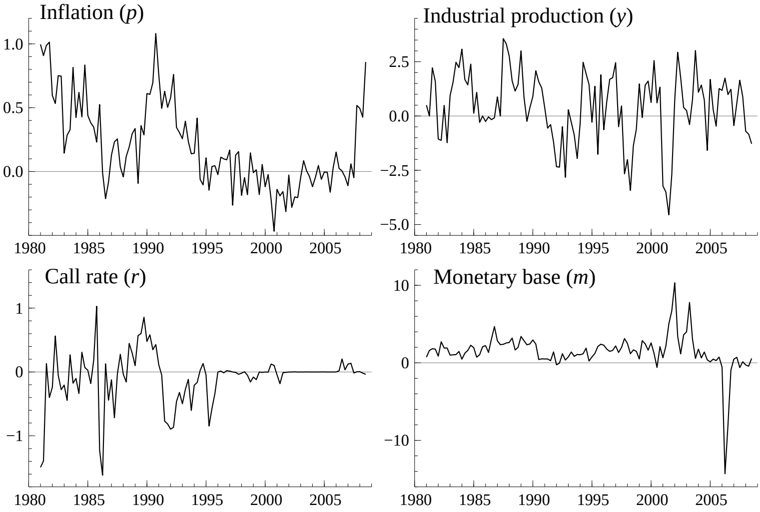
- 1981/1Q–2008/3Q, quarterly

Transformation:

- $r \to$ first difference
- $p, y, m \to$ first log difference

Application to the Japanese Economy

Japanese macroeconomic data (1981/1Q to 2008/3Q).



Application to the Japanese Economy

Priors

$$V_\beta \sim IW(25, 0.01), v_{ai}^2 \sim IG(4, 0.02), v_{hi}^2 \sim IG(4, 0.02)$$

Inittial state of the time-varying parameters

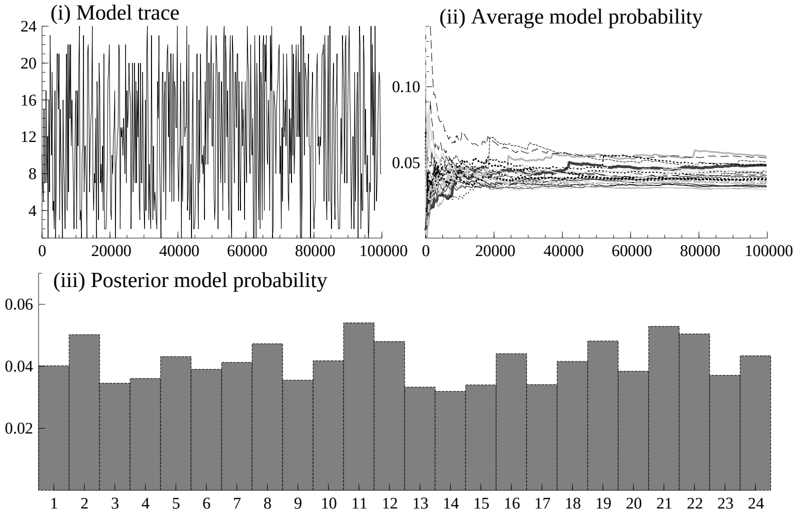
$$\beta_{s+1} \sim N(0, 10I), a_{s+1} \sim N(0, 10I), h_{s+1} \sim N(0, 50I)$$

Details

- The lag-length of TVP-VAR model is set two.
- We generate 100,000 draws after the initial 10,000 draws are discarded as burn-in.
- MH acceptance probability is 29.7%.

Application to the Japanese Economy

Estimation results of the RJMCMC algorithm



Application to the Japanese Economy

- Full sample: 1981/1Q–2008/3Q.
- Subsamples: 1981/1Q–1995/4Q and 1996/1Q–2008/3Q
- After the Bank of Japan lowered the official discount rate from 1.0% to 0.5% in September 1995, the overnight call rate stayed in the very low level during the second subsample period until the raising of the target overnight call rate to 0.25% from the zero interest rate policy in July 2006.

Application to the Japanese Economy

Posterior model probability: top 3 models

	Full sample		Sub samples			
	1981-2008		(i)1981-1995		(ii)1996-2008	
Rank	Order	Prob.	Order	Prob.	Order	Prob.
1	(y, m, p, r)	0.054	(y, r, m, p)	0.068	(m, y, p, r)	0.071
2	(m, y, p, r)	0.053	(r, y, m, p)	0.066	(m, y, r, p)	0.048
3	(m, y, r, p)	0.050	(y, m, r, p)	0.052	(p, y, m, r)	0.046

Application to the Japanese Economy

Model choice

- Posterior odds ratio
$$\frac{p(\mathrm{M}_i|\boldsymbol{y})}{p(\mathrm{M}_j|\boldsymbol{y})} = \frac{p(\boldsymbol{y}|\mathrm{M}_i)}{p(\boldsymbol{y}|\mathrm{M}_j)}\frac{p(\mathrm{M}_i)}{p(\mathrm{M}_j)}$$
- $p(\boldsymbol{y}|\mathrm{M}_i)/p(\boldsymbol{y}|\mathrm{M}_j)$: Bayes factor
- $p(\mathrm{M}_i)/p(\mathrm{M}_j)$: Prior odds ratio
- $p(\boldsymbol{y}|\mathrm{M}_i)$ or $p(\boldsymbol{y}|\mathrm{M}_j)$: Marginal likelihood
- $p(\boldsymbol{y}|\mathrm{M}_i)/p(\boldsymbol{y}|\mathrm{M}_j) > 1 \rightarrow$ Choose M_i.
- If $p(\mathrm{M}_i)/p(\mathrm{M}_j) = 1$,
$$\frac{p(\mathrm{M}_i|\boldsymbol{y})}{p(\mathrm{M}_j|\boldsymbol{y})} = \frac{p(\boldsymbol{y}|\mathrm{M}_i)}{p(\boldsymbol{y}|\mathrm{M}_j)}$$
- Then, the model whose marginal likelihood is larger is chosen.

Application to the Japanese Economy

Model choice

- It is straightforward to calculate marginal likelihood using the harmonic mean method proposed by Geweke (1999).

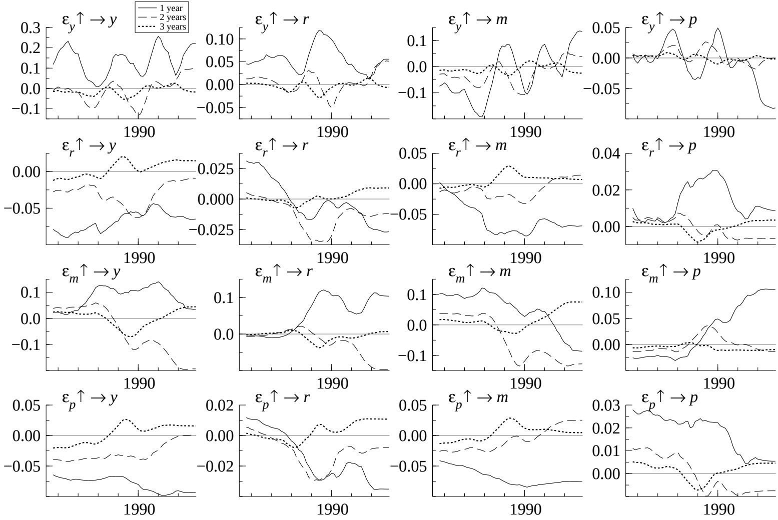
Application to the Japanese Economy

Estimated marginal likelihoods (ML) in logarithm scale. TVP refers to: time-varying parameter model, STVP: semi time-varying parameter model (1: time-varying h_t, 2: time-varying β_t and a_t), CP: constant parameter model.

	Full sample	Subsamples	
	1981-2008	1981-1995	1996-2008
(a) Under the ordering uncertainty (RJMCMC)			
TVP	-434.62	-264.56	-385.99
(b) Top model			
TVP	-291.91	-256.89	-214.11
STVP1	-360.24	-261.49	-238.61
STVP2	-498.25	-264.44	-280.66
CP	-544.54	-288.02	-256.81

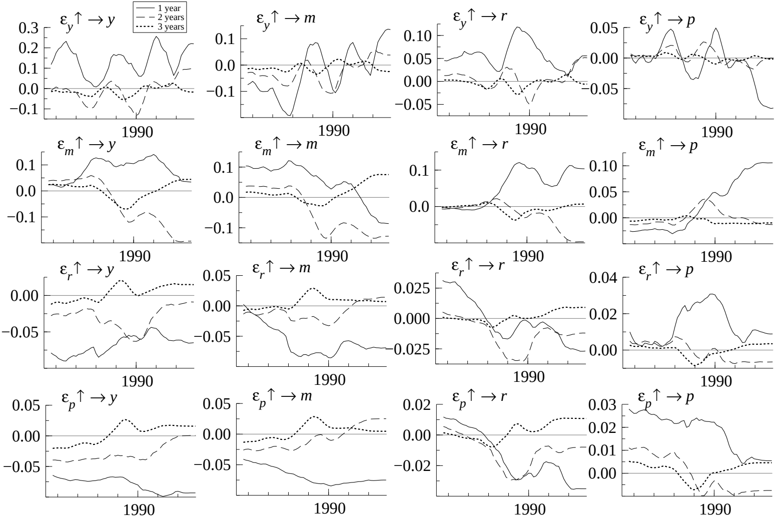
Application to the Japanese Economy

IRF: (y, r, m, p) which is the top ordering in the first subsample (1981–1995)



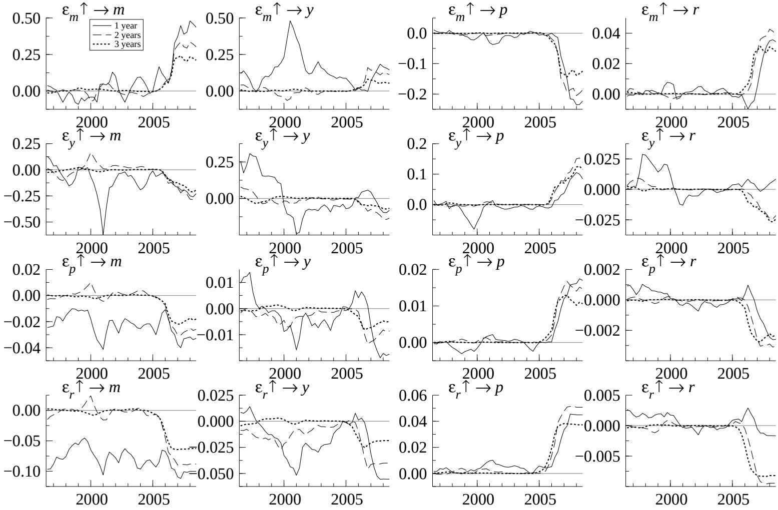
Application to the Japanese Economy

IRF: (y, m, p, r) which is the top ordering in the full sample (1981–1995)



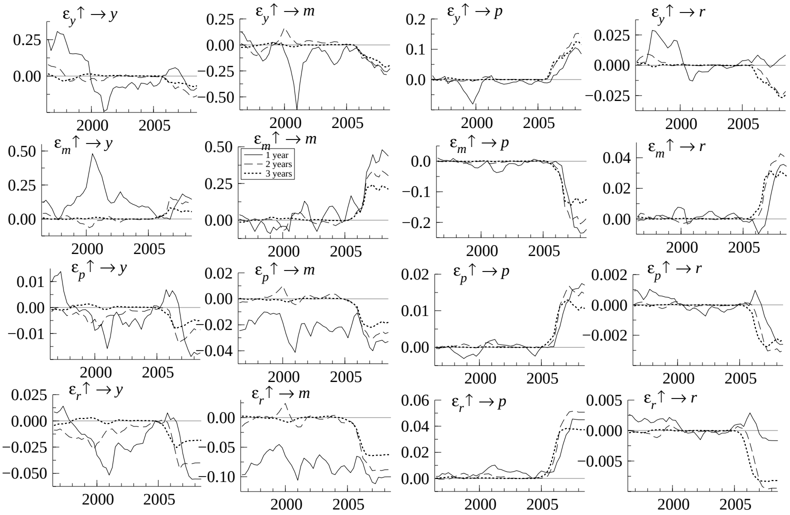
Application to the Japanese Economy

IRF: (m, y, p, r) which is the top ordering in the second subsample (1996–2008)



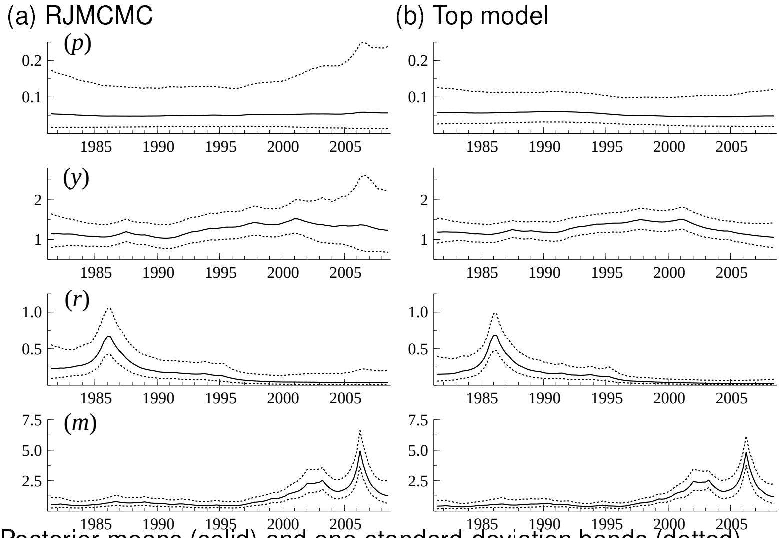
Application to the Japanese Economy

IRF: (y, m, p, r) which is the top ordering in the full sample (1996–2008)



Application to the Japanese Economy

Estimates of stochastic volatility $\sigma_{it} = \exp(h_{it}/2)$



Posterior means (solid) and one-standard-deviation bands (dotted).

Simulation Study 1

Simulate observations

- TVP-VAR model: $n = 120$, $k = 3$, $y_t = (y_{1t}, y_{2t}, y_{3t})$
- Recursive structure for A_t
- Standard deviations of the errors for the process of β_t, h_t and a_t:

  $v_{\beta i} = 0.005, \quad v_{hi} = 0.1$

  Compare (i) $v_{ai} = 1.0$, and (ii) $v_{ai} = 0.1$

Key: the source of ordering selection is the variation of a_t

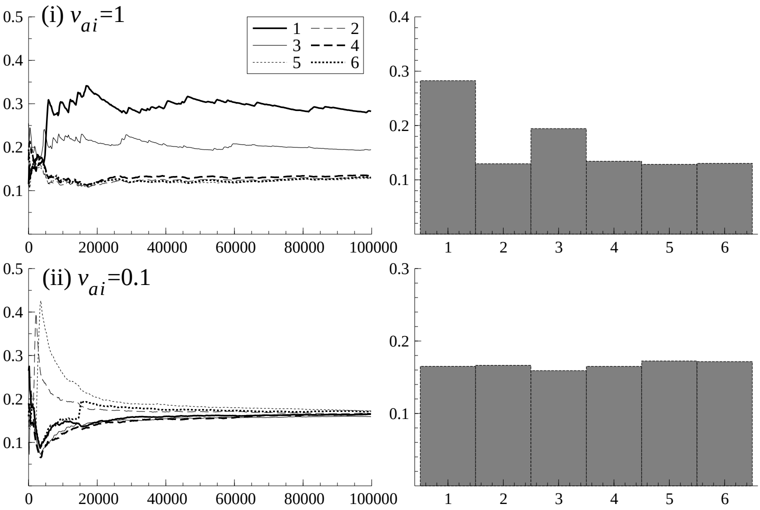
Simulation Study 1

Model ordering for RJMCMC

No.	Order
1	(y_{1t}, y_{2t}, y_{3t}) (True)
2	(y_{1t}, y_{3t}, y_{2t})
3	(y_{2t}, y_{1t}, y_{3t})
4	(y_{2t}, y_{3t}, y_{1t})
5	(y_{3t}, y_{1t}, y_{2t})
6	(y_{3t}, y_{2t}, y_{1t})

Simulation Study 1

Results: trace plots and posterior model probability



Simulation Study 2

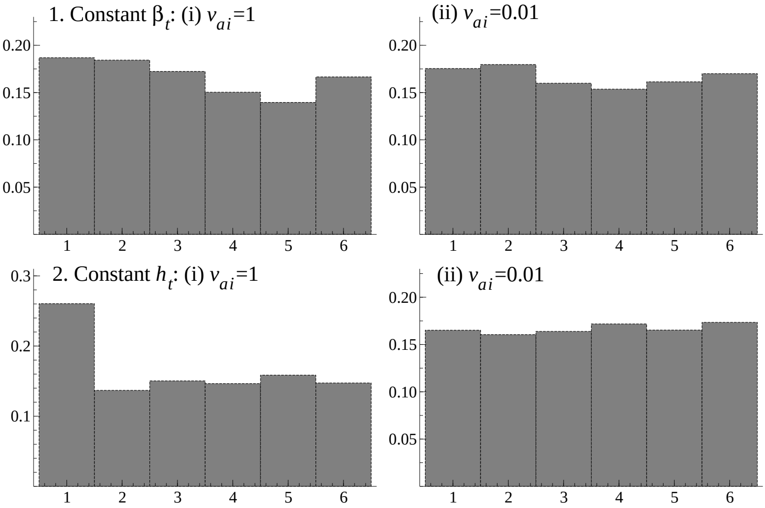
Simulate observations

1. Constant coefficient: $\beta_t \equiv \beta$ (time-varying h_t)
2. Constant volatility: $h_t \equiv h$ (time-varying b_t)

- Data is simulated from (and RJMCMC is implemented for) these partially time-varying models
- Time-varying a_t: source of ordering
- Compare (i) $v_{ai} = 1.0$, and (ii) $v_{ai} = 0.1$

Simulation Study 2

Results: posterior model probability



Simulation Study 2

Results

1. Constant coefficient: $\beta_t \equiv \beta$
   $\rightarrow$ Time-varying a_t and h_t tend to capture more variation than needed, which makes model selection difficult.

2. Constant volatility: $h_t \equiv h$
   $\rightarrow$ Time-varying a_t may capture some (unessential) variation of volatility, though RJMCMC can detect the true model.

Example of Different Identification

Sims and Zha (1996)

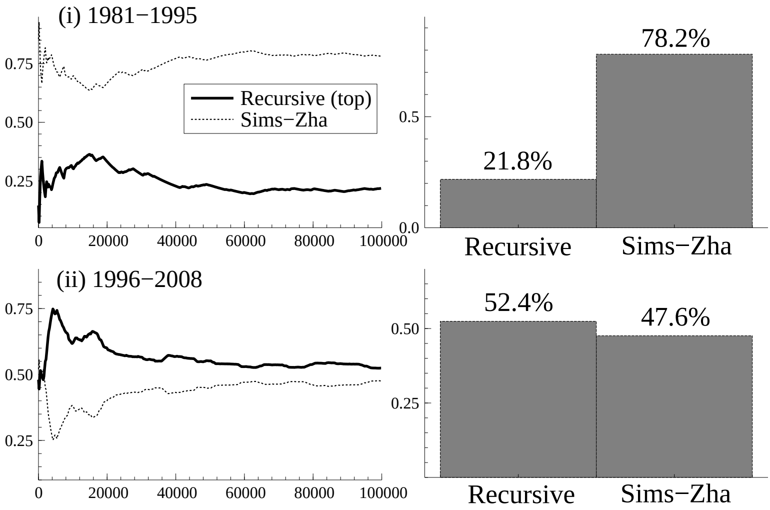
$$A_t y_t = \begin{pmatrix} 1 & 0 & 0 & 0 \\ a_{21,t} & 1 & 0 & 0 \\ 0 & 0 & 1 & a_{34,t} \\ a_{41,t} & a_{42,t} & a_{43,t} & 1 \end{pmatrix} \begin{pmatrix} y_t \\ p_t \\ r_t \\ m_t \end{pmatrix}$$

RJMCMC for

1. Recursive identification: top ordering (y_t, m_t, p_t, r_t)
2. Sims-Zha model

Example of Different Identification

Results: trace plot and posterior model probability



Conclusions

This paper

- proposes a method for the ordering of variables in the TVP-VAR model using the RJMCMC.
- shows using simulation that whether our method works well or not depends on how variable the parameters a_t and β_t are.
- applies it to the Japanese macroeconomic data.

Conclusions

Empirical results

- The TVP-VAR model is favored over the models where the coefficients, volatilities and the both are constant.
- Fixing the order as the one which has the highest posterior probability is favored over taking account of order uncertainty.
- The introduction of zero interest rate policy may have changed the order of variables.
- The Sims and Zha (1996) identification is favored over the recursive identification before the zero interest rate policy, which is reversed after the introduction of zero interest rate policy.

Future Work

1. Other identifying restrictions, e.g., sign restriction, DSGE-VAR
2. Zero bound on nominal interest rate (Iwata and Wu, 2006; Nakajima, 2011)