

A Study of Smoothing Methods on Small Area Population Projection



Shuoh Jin

Jack C. Yue

National Chengchi University

Dec. 18, 2011



Summary

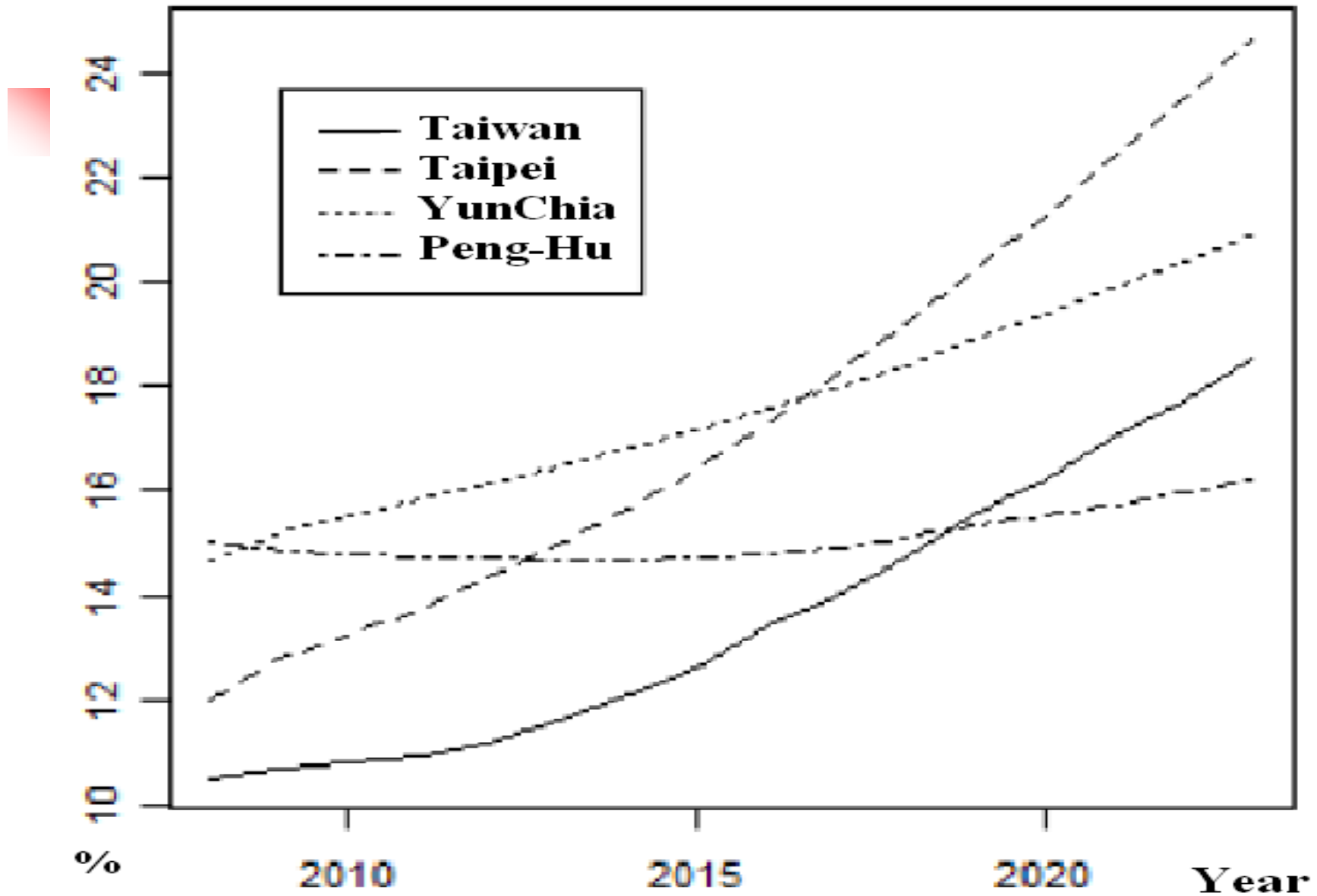
- Motivation
- Graduation Methods
- Simulation Studies
- Empirical Simulation
- Conclusion & Discussions



Population Aging in Taiwan

- The population aging is a common phenomenon in Taiwan, but the problem is not the same for different counties.
 - The population aging in Taipei city is faster.
 - Small area population projection is essential.
- It is difficult to derive a reliable projection for small areas, and even more difficult if the social attributes within the small areas vary widely.

Proportion of Ages 65+ in Taiwan (unit: %)





Important Factors

- Difficulties for small area population projection can be attributed into three directions:
 - Population size;
 - Number of base years;
 - Projection horizon.

Note: Smith and Sincich (1990) studied the influences of “base year” and “projection horizon” for simple projection methods (e.g. linear & exponential) on large populations.



Study Objective

- Use smoothing techniques to improve the stability and accuracy of small area population projection.
 - Consider Cohort Component method.
- Basically, smaller population size would have larger projection errors.
 - e.g., We use the projection of life expectancy in Taipei city as a demonstration.

Life Expectancy Projection in Taipei City



Note: The projection is based on 1,000 block bootstrap simulation runs.

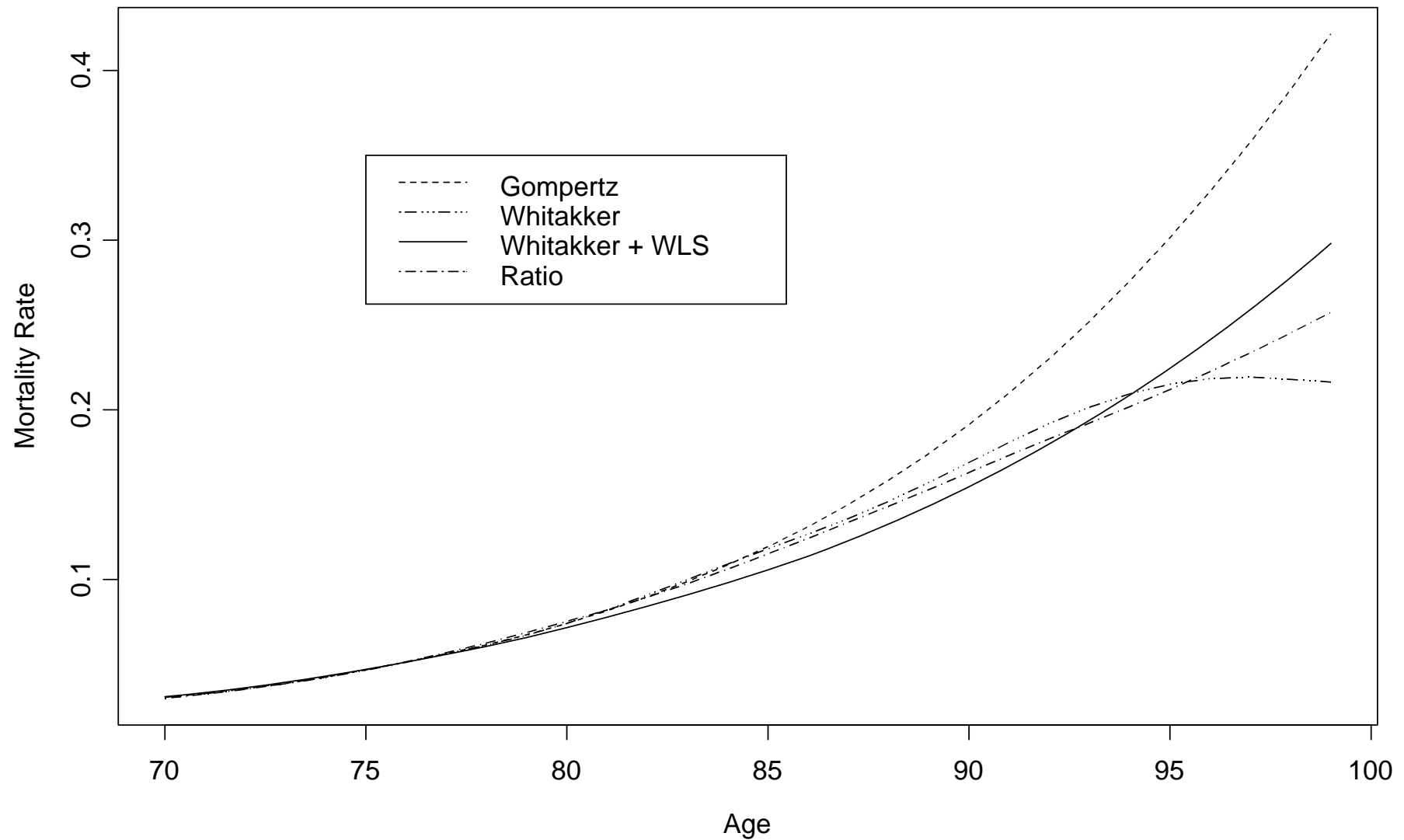


What is Graduation ?

- *Graduation may be defined as the process of obtaining from irregular set of observed values of a dependent variable a corresponding smooth set of values consistent in a general way with the observed values.*

Note: Graduation will possibly alter the original data properties and it should be applied with care.

Taiwan Male Mortality Rates (1999-2001)



Graduation Methods on Taiwan Elderly Mortality



Graduation (Smoothing) Methods

- Graduation has been studied for more than one century. Mathematical formulas were popular in early days (e.g., moving average) but they often suffer to the weight problem.
- Methods incorporating weight are considered:
 - Whittaker (Henderson) method
 - Mortality Ratio (Whittaker) method
 - Partial SMR (Lee, 2003)



Graduation Methods (cont.)

- Whittaker-Henderson method:

$$M = F + hS = \sum_{x=1}^n w_x (v_x - u_x)^2 + h \sum_{x=1}^{n-z} (\Delta^z v_x)^2$$

→ The idea of Whittaker method is similar to the Penalized Likelihood Estimation (PLE), by setting the objective function to

$$L(\theta | data) + \frac{\lambda}{2} J(\theta) \quad \text{or} \quad \sum_{i=1}^n \left(\frac{Y_i - \theta(x_i)}{\sigma_i} \right)^2 + \frac{\lambda}{2} \int \ddot{\theta}(x)^2 dx$$



Graduation Methods (cont.)

- Mortality Ratio (Whittaker) method:

Applying Whittaker on mortality ratio $r_x = \frac{u_x}{u_x^*}$,

u_x^* is the mortality rates from the reference population.

Note: u_x & v_x are the raw & graduated rates.



Graduation Methods (cont.)

- Partial SMR (Standard Mortality Ratio)

$$v_x = u_x^* \times \exp \left(\frac{d_x \times \hat{h}^2 \times \log(d_x / (n_x \cdot u_x^*)) + (1 - d_x / \sum d_x) \times \log(\text{SMR})}{d_x \times \hat{h}^2 + (1 - d_x / \sum d_x)} \right)$$

where

$$\text{SMR} = \frac{\sum_x d_x}{\sum_x n_x \cdot u_x^*}$$

$$h^2 = \max \left(\frac{\sum \left((d_x - n_x \times u_x^* \times \text{SMR})^2 - \sum d_x \right)}{\text{SMR}^2 \times \sum (n_x \times u_x^*)^2}, 0 \right)$$



Cohort Component Method

- Cohort component method is the most popular population projection method.

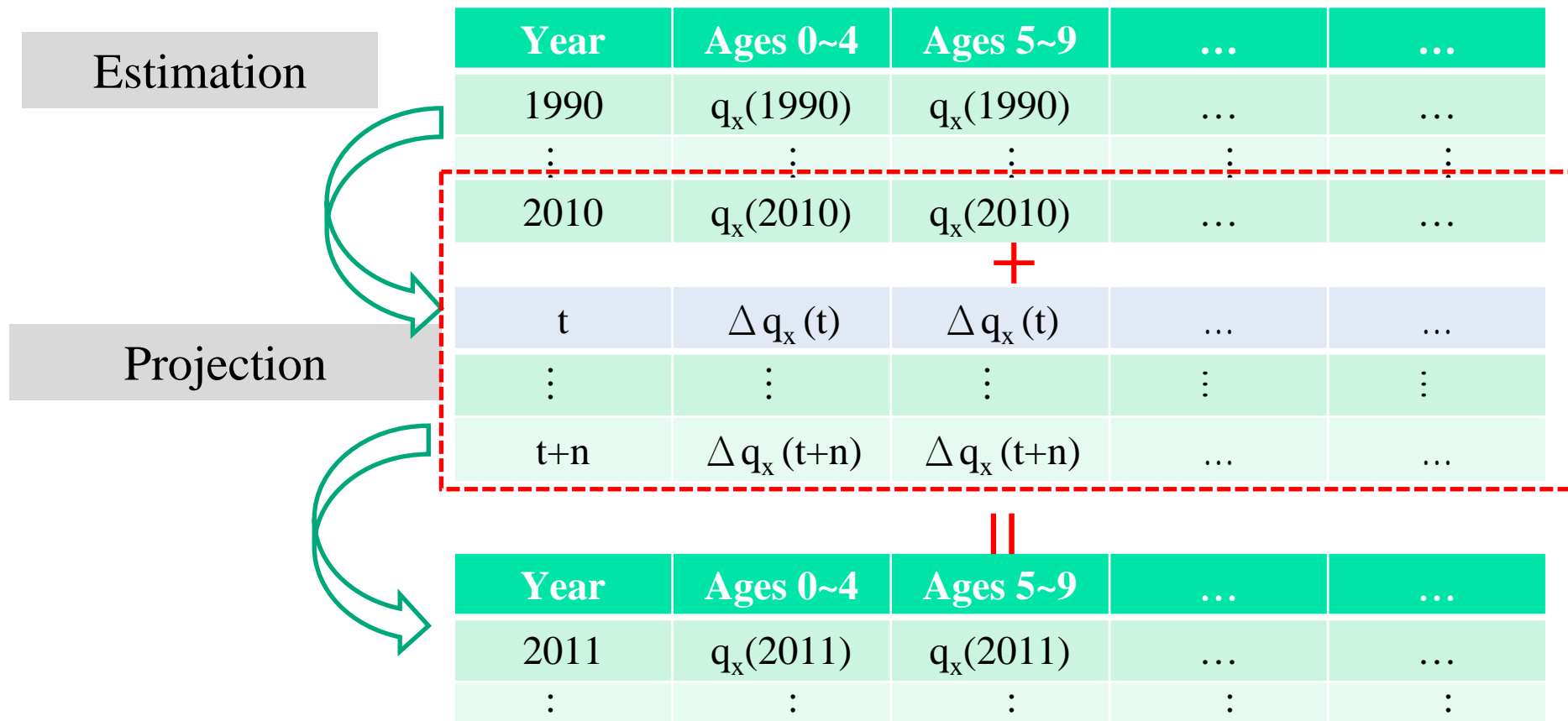
→ Use the results at year t to project for year $t+1$,
 i.e., $P(t+1) = S(t) \times P(t) + I(t) - E(t)$

where $P(t) = [{}_tP_0, {}_tP_1, \dots, {}_tP_\omega]^T$ and

$$S(t) = \begin{bmatrix} 0 & 0 & \dots & 0 & S_t(\alpha, \alpha+0.5) \cdot f_\alpha^t & \dots & S_t(\beta, \beta+0.5) \cdot f_\beta^t & 0 & \dots & 0 & 0 \\ S_t(0,1) & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & S_t(1,2) & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & S_t(w-1,w) & 0 \end{bmatrix}$$

Simulation-based Projection Method

- Use **Block Bootstrap** to project fertility, mortality, migration.





Lee-Carter Mortality Model

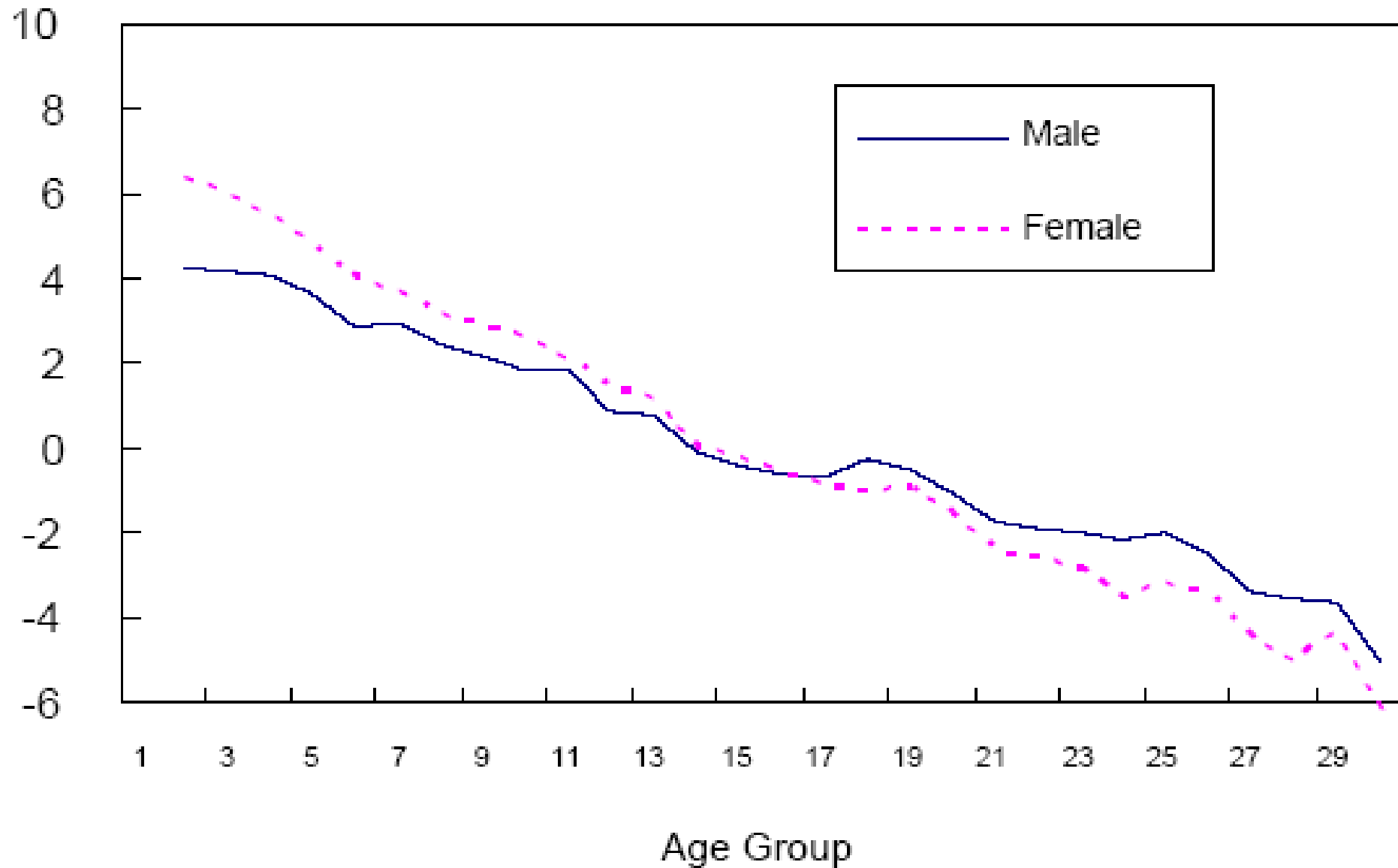
- The Lee-Carter Model (Lee and Carter, 1992), the central mortality rate should be consistent with the following equation

→

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}$$

α_x describes the average age-specific mortality, κ_t represents the general mortality level, and the decline in mortality at age x is captured by β_x .

κ_t Estimates of Lee-Carter Model in Taiwan



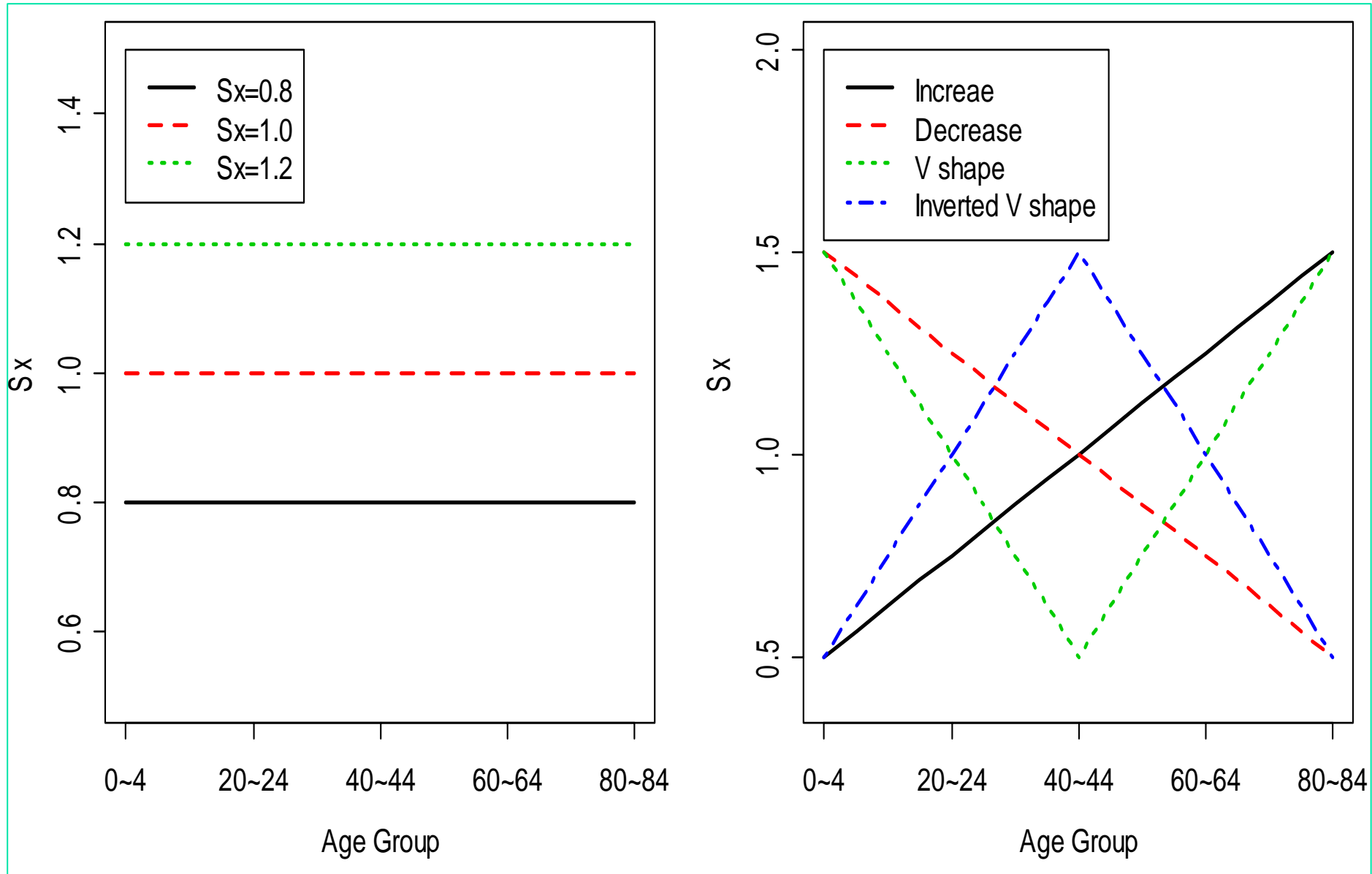


Simulation Setting

- The reference population is larger than the small population, and the mortality rates of reference population satisfy the LC model.
- The mortality rates of small population follow one of 7 mortality scenarios:
 - Similar to the reference group (3 cases)
 - Differ to the reference group (4 cases)

Note: We use mortality ratio $s_x = \frac{q_x}{q_x^*}$ to measure.

Seven Mortality Scenario





Simulation Setting (cont.)

- Taiwan is the reference population and counties in Taiwan are the small populations.
 - 5-age group (0-4, 5-9, ..., 80-84)
 - Training vs. Testing Periods
- Comparison criterion:

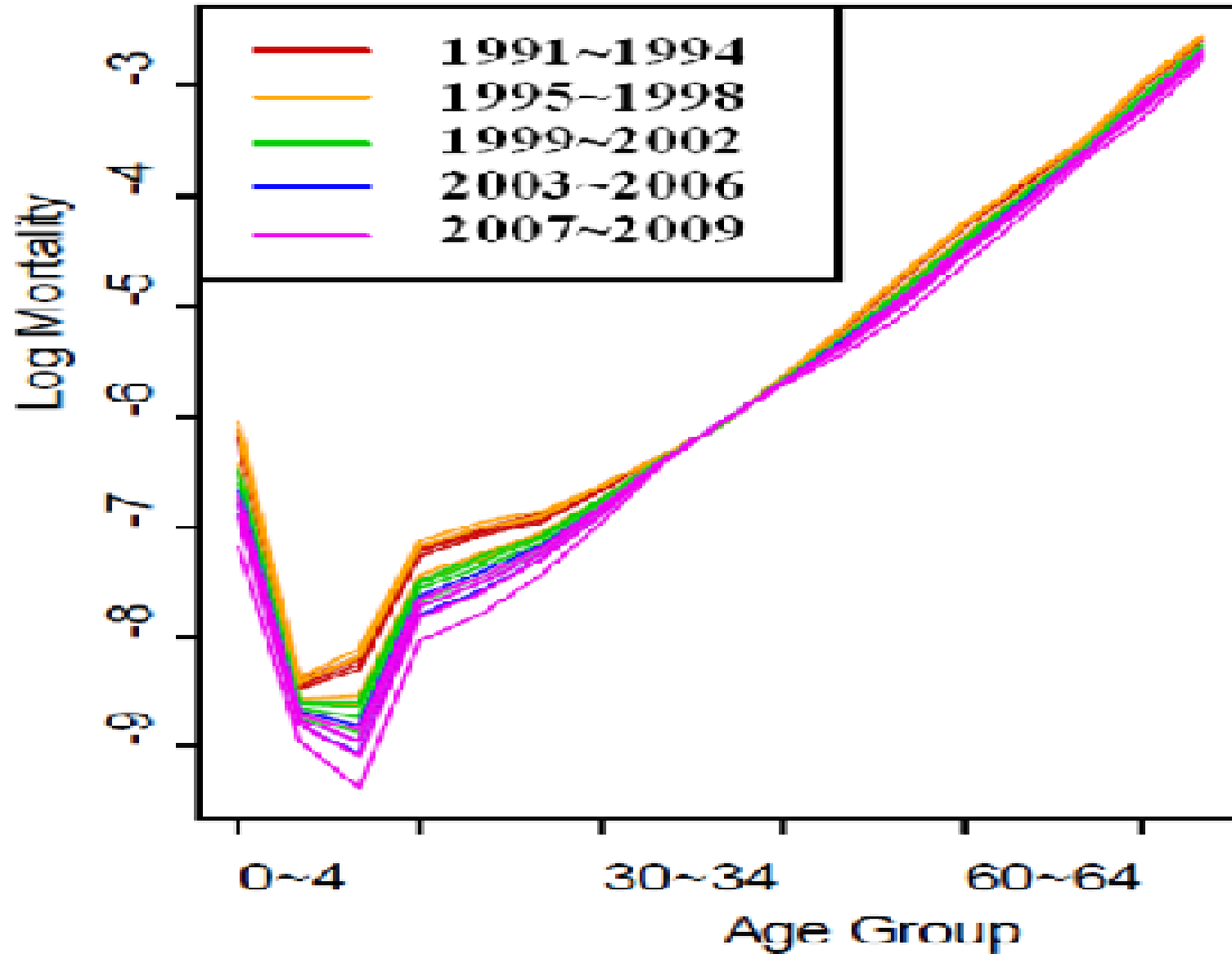
$$\text{MAPE} = \frac{1}{n} \sum_{t=1}^n \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| \times 100\%$$



Simulation Study 1

- The first part of simulation is to check if the graduation methods can reduce the variance.
 - The population of a sub-county in Taipei city (Xin-Yi Township), around 0.22 million, is treated as the target small population.
- Taipei city (2.6 million) is the reference and its mortality rates are assumed to follow the Lee-Carter model.
- 1,000 simulation runs (1991~2009)

Lee-Carter Mortality Rates (1991~2009)



Simulation Errors of Graduating Sub-county Data

MAPE (%)

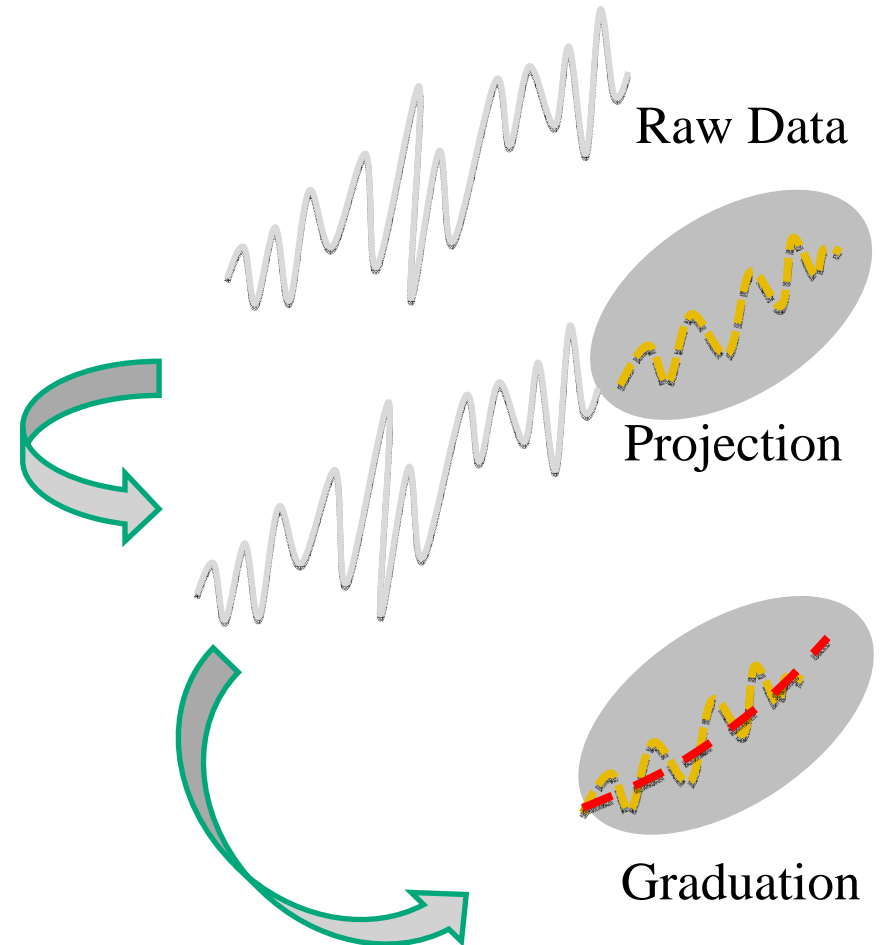
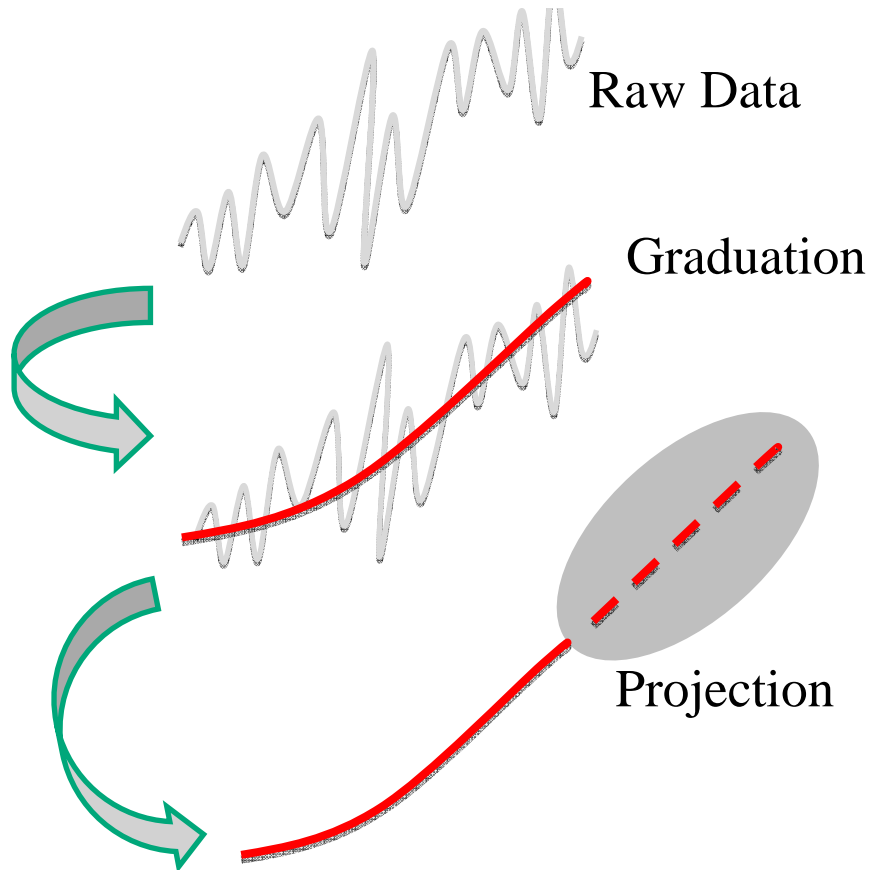
	80%	100%	120%	Incr.	Decr.	V	Λ
Raw	40.8	36.7	33.1	42.5	33.4	35.2	39.6
Whittaker	31.7	29.6	27.9	32.9	28.4	31.7	29.8
<input checked="" type="checkbox"/> Partial SMR	4.3	3.8	3.6	46.7	25.6	25.5	18.5
<input checked="" type="checkbox"/> Whittaker Ratio	12.7	12.7	12.0	32.8	22.3	13.5	16.0

The Order of Projection (which first?)

Graduation → Projection

Projection → Graduation

■ **Easier to handle**





Simulation Study 2

- The second part of simulation is to check if the graduation or projection shall go first.
 - Partial SMR & Whittaker Ratio
 - Suppose the Lee-Carter model is true
- Data: Training (1990~2004) & Testing (2005~2009; Block Bootstrap). Taipei city (2.6 million) is the reference and Xin-Yi Township (0.22 million) is the target .
 - 7 s_x scenarios are considered



Simulation Errors of Projection Order

	MAPE (%)						
	80%	100%	120%	Incr.	Decr.	V	Λ
W/O Graduation	42.4	41.1	36.6	39.4	36.3	39.4	36.2
Partial SMR G→P	16.9	16.8	16.3	37.5	30.0	27.5	22.9
Whittaker Ratio G→P	24.2	23.2	21.9	28.8	27.7	23.6	24.4
Partial SMR P→G	16.1	15.7	15.1	36.0	30.1	28.1	23.6
Whittaker Ratio P→G	21.1	20.2	20.1	24.6	28.5	22.7	23.3

Note: the projection order doesn't have big influence.

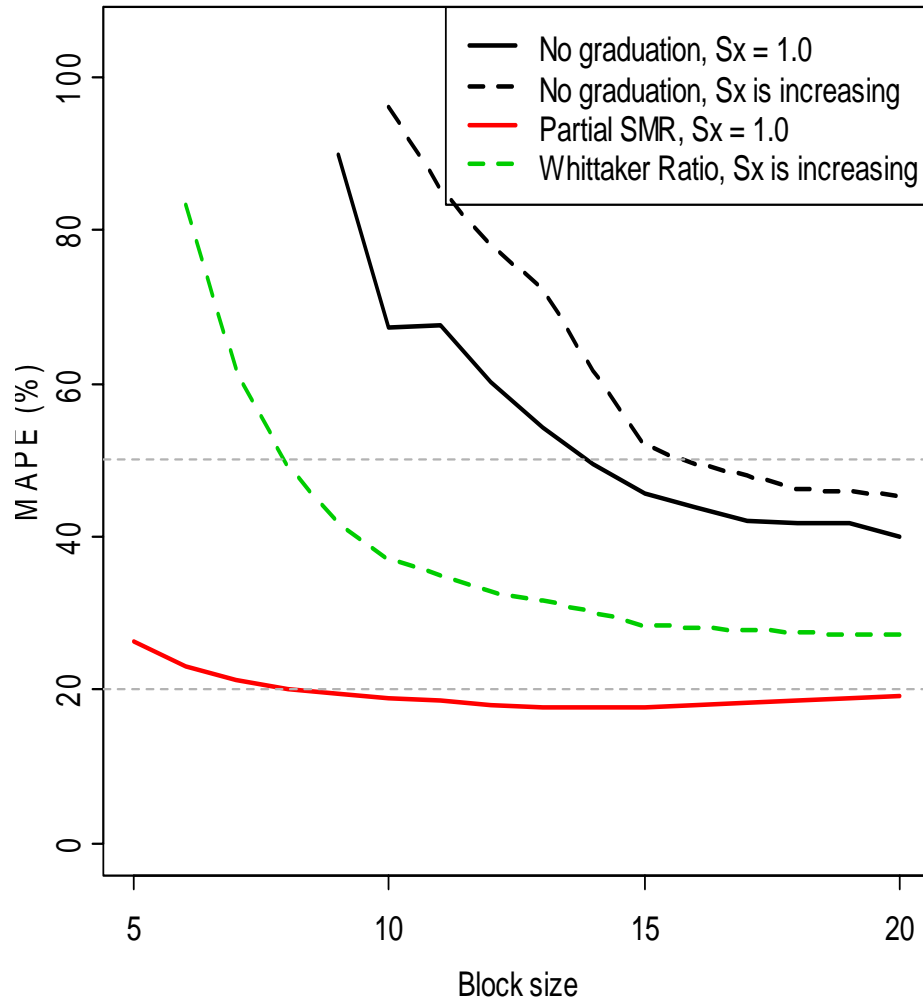


Simulation Study 3

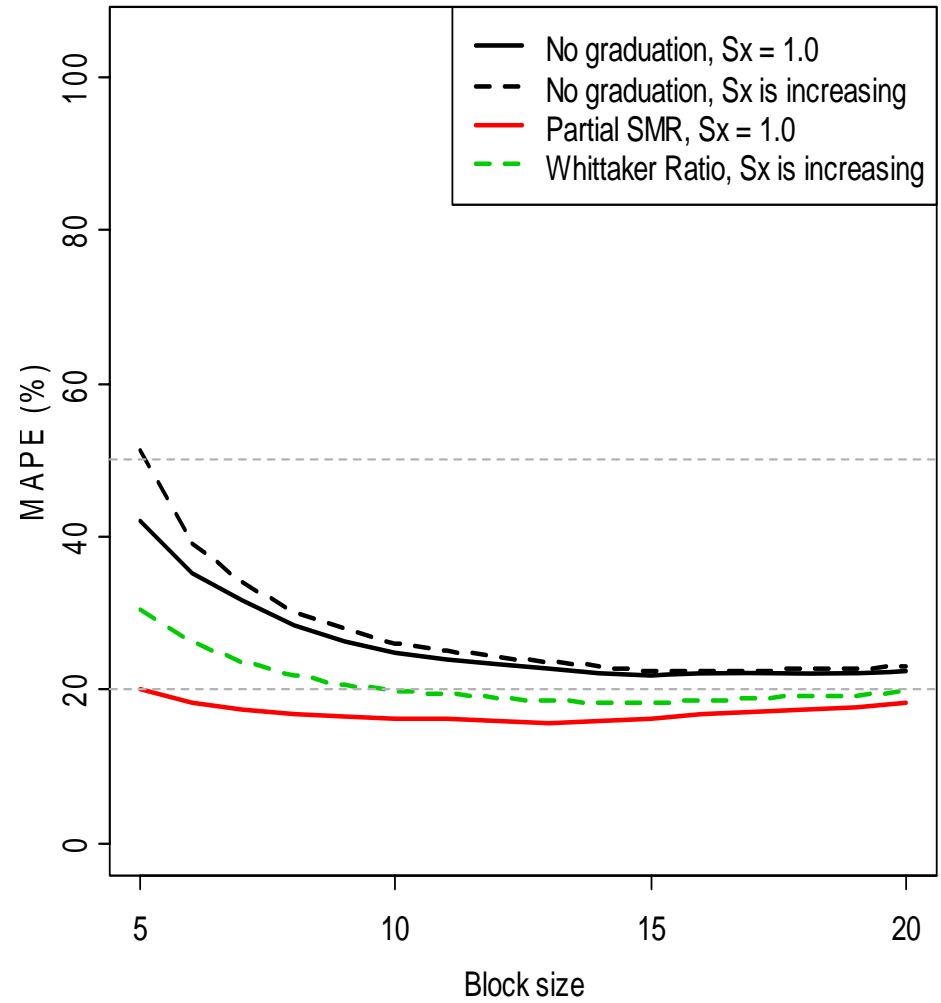
- The third part of simulation is to evaluate the influence of “Block size,” “Population size,” “Base year,” and “Projection horizon”.
 - Partial SMR & Whittaker Ratio
- Data: Training (1950~1979) & Testing (1980~2009; Block Bootstrap). Taiwan population (23 million) is the reference.
 - $S_x = 1$ & Increasing S_x is considered
 - Several sizes of small population

Influence of Block Bootstrap Size

Population 100,000

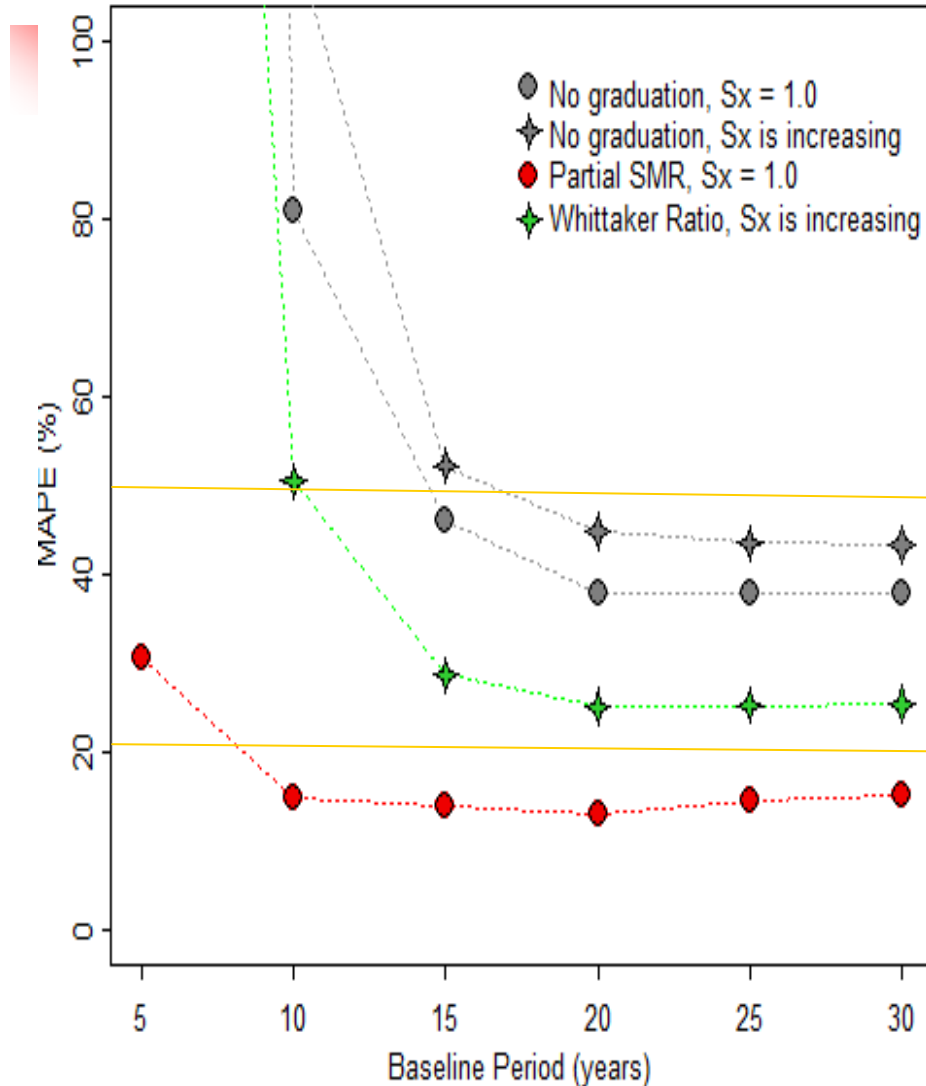


Population 500,000

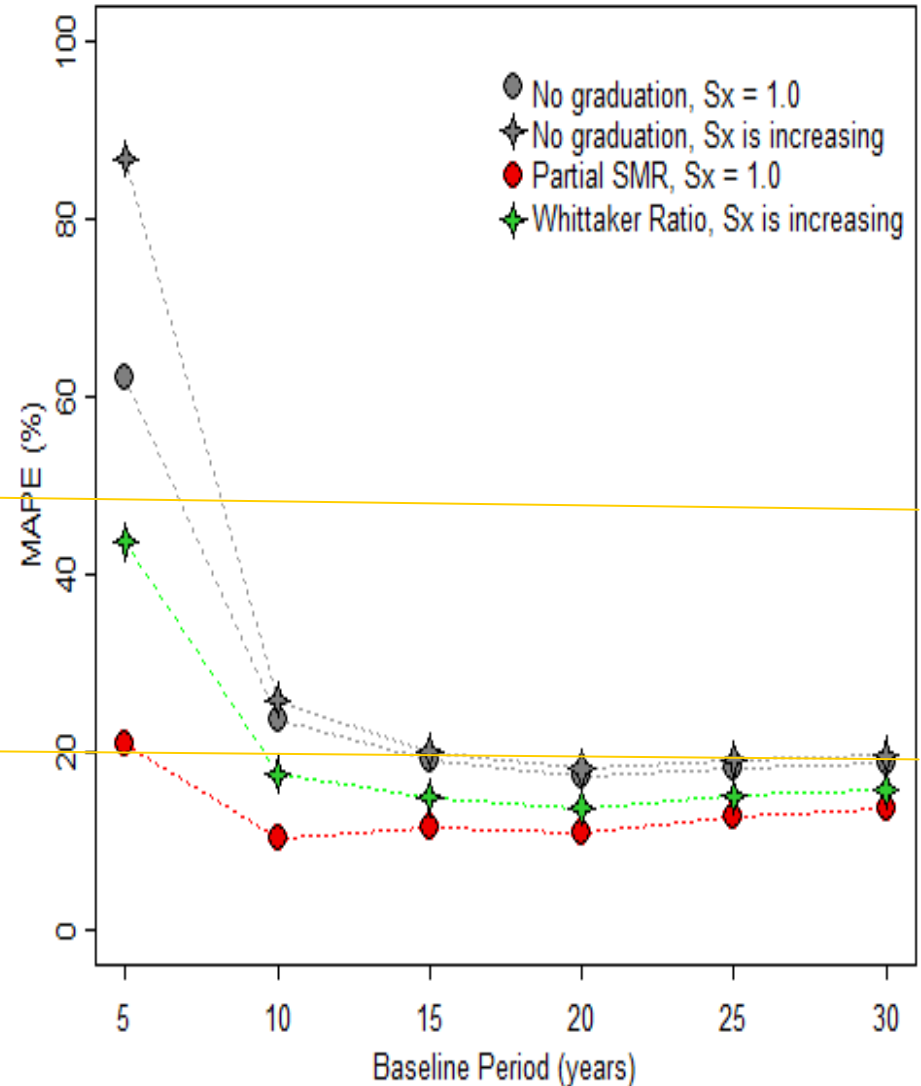


Influence of Length of Base Years

Population 100,000

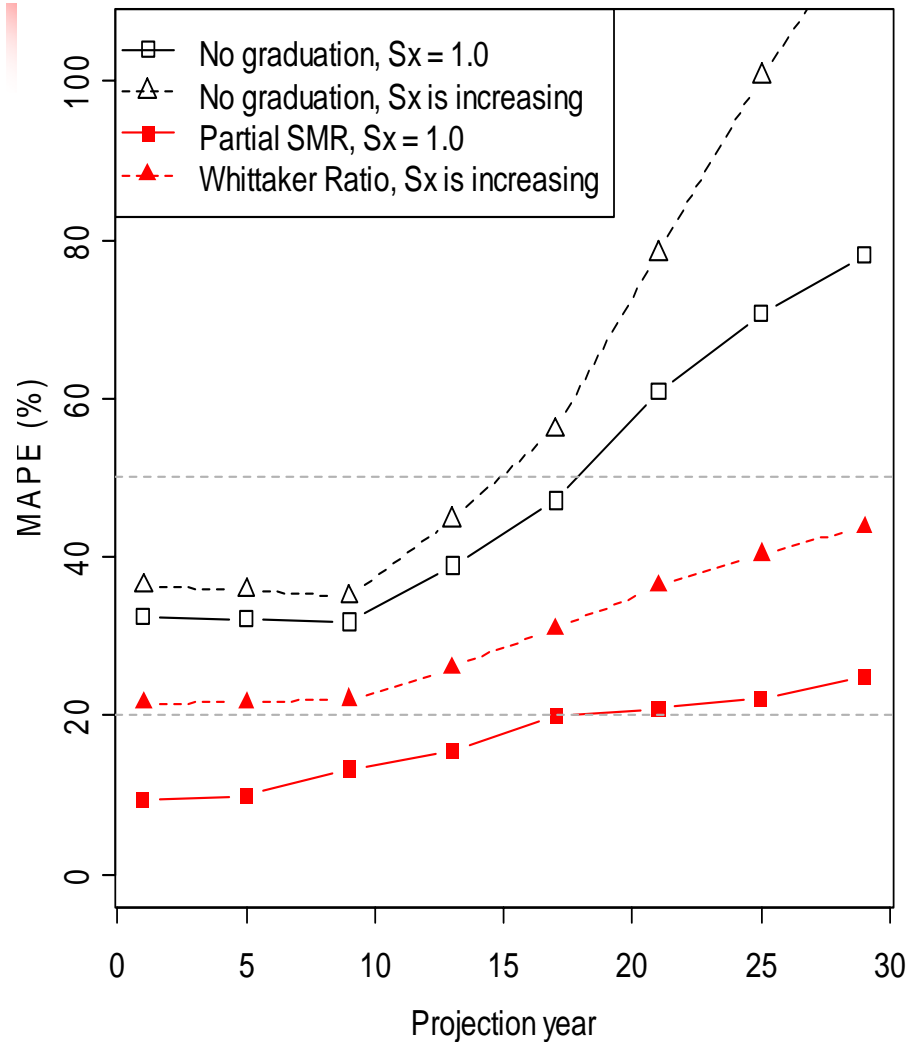


Population 500,000

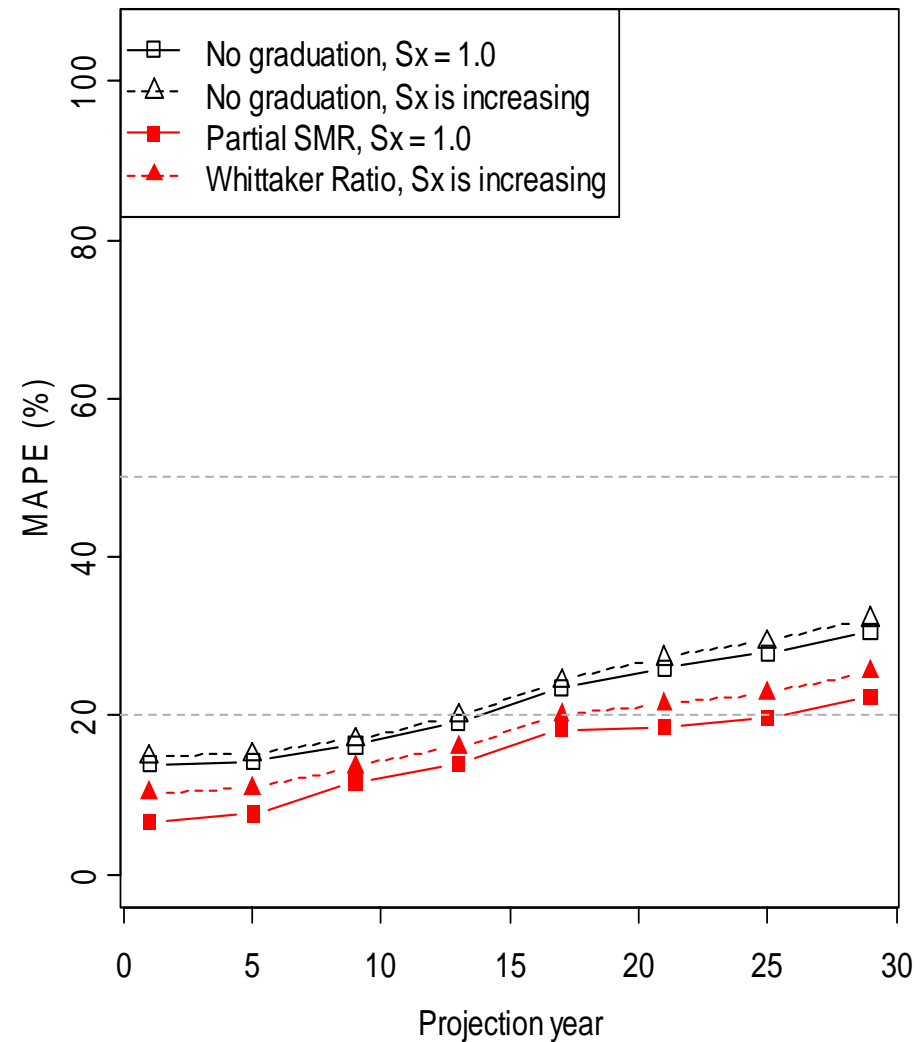


Influence of Projection Horizon

Population 100,000



Population 500,000

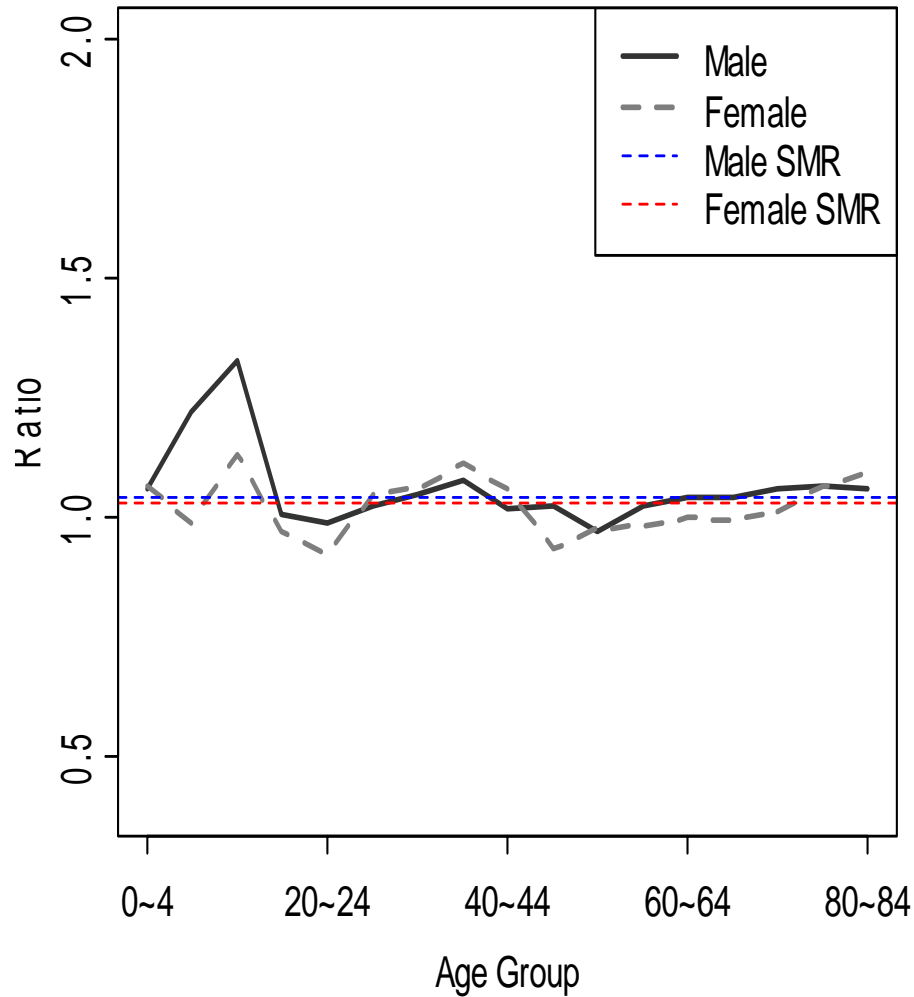




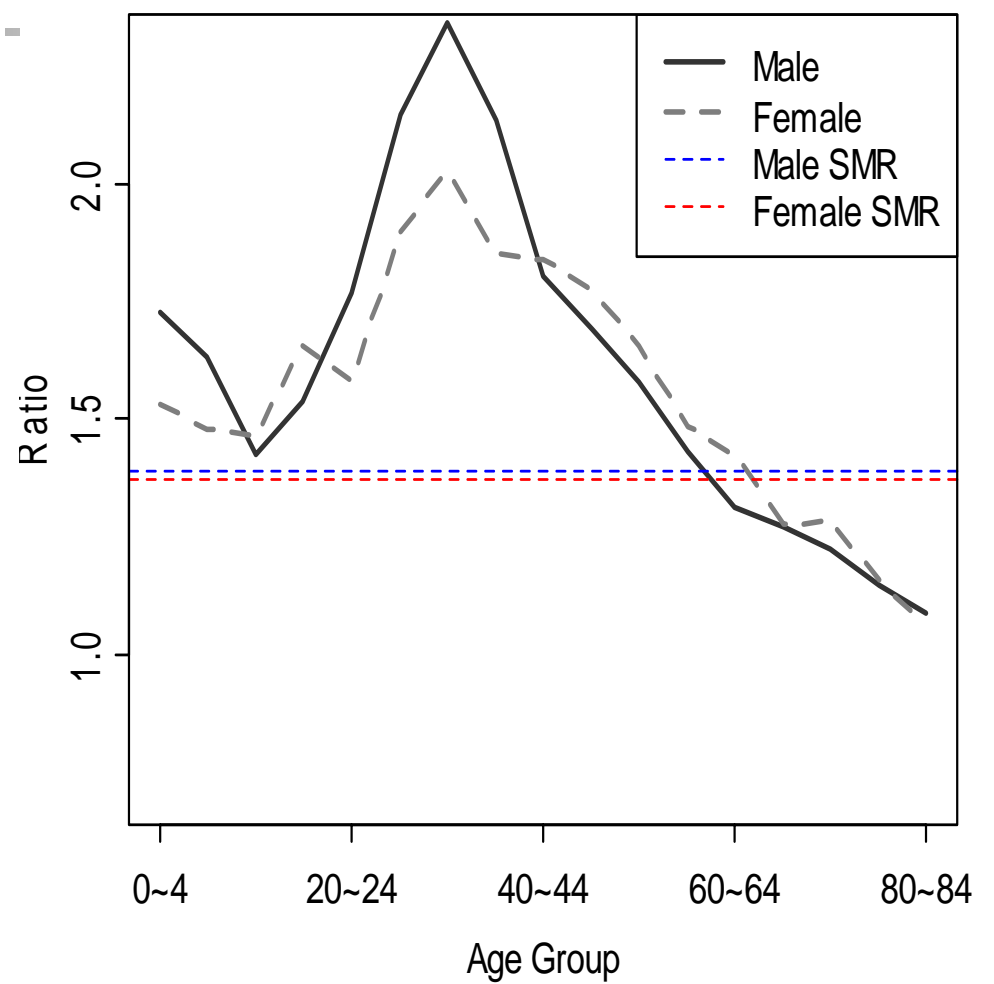
Empirical Study

- We also conduct the population projection of 2 small counties in Taiwan (0.20 & 1.0 million). Taiwan population (23 million) is the reference.
 - Training period: 1980~1994;
Testing period: 1995~2009
 - 5-age group (0-4, 5-9, ..., 80-84)

Tainan County
(1.0 million)



Taitung County
(0.2 million)





Prediction Errors of Empirical Simulation

MAPE (%)

	Tainan County		Taitung County	
	Male	Female	Male	Female
W/O Graduation	43.5	43.9	70.0	102.4
Partial SMR	26.7	27.2	45.6	48.4
Whittaker Ratio	27.8	36.0	44.0	57.8



Conclusion

- More needs for small area population projection and the population size is a key factor.
- We propose using graduation to reduce the projection errors for small areas.
 - The methods of mortality ratio & partial SMR can reduce the variance of prediction.
 - The degree of influence on projection:
Population Size > Base Years > Projection Horizon
 - Base years ≥ 15 & Projection Horizon ≤ 20 .



Discussions

- About small area graduation:
 - Include age-period-cohort (APC) relationship between small & reference areas in Partial SMR
 - 2-dim or 3-dim (APC) Whittaker Ratio
 - Other methods (e.g., Bayesian or parametric)
- Apply graduation on a group of small populations, say, a group of heterogeneous populations?
- Combine graduation & multi-area migration model.



Thank you
for your attention!

