√ "A Broad Class Estimators for Two Ordered Normal Means with Ordered Variances under Pitman's Comparison"

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Introduction

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- in recent two monographs, Silvapulle and Sen (2004) and Van Eeden (2006).

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- √ We discuss the estimation of two ordered means, individually and/or simultaneously, under Pitman closeness criterion.

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$$P_r\{|T_1 - \theta| < |T_2 - \theta|\} > 1/2.$$

About Pitman closeness criterion

- √ refer the reader to Rao (1981), Keating and Manson (1985), Peddada (1985), Rao, keating and Mason (1986), Khattree and Peddada (1987) for details.

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- were unified in the monograph by Keating, Manson and Sen (1993).

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- Here we consider the estimation of two ordered normal means when unknown variances are ordered using Pitman closeness criterion.
- We propose the estimators which is closer to the unknown means than the usual estimators which ignore the order restriction on variances using the modified Pitman closeness criterion suggested by Gupta and Singh (1992).

The history background

- First, we state some fundamental results on the estimation of common mean and ordered means when the MSE or stochastic domination is concerned.

the unbiased estimators of μ_i and σ_i^2,

- ✓ Let $X_{ij}, i = 1, 2, j = 1, \ldots, n_i$ be independent observations from normal distribution with mean μ_i and variance σ_i^2, where both μ_i and σ_i^2 are unknown.

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- Let $X_{ij}, i = 1, 2, j = 1, \ldots, n_i$ be independent observations from normal distribution with mean μ_i and variance σ_i^2, where both μ_i and σ_i^2 are unknown.
- Also let

$$\bar{X}_i = \sum_{j=1}^{n_i} X_{ij}/n_i \,\text{and}\, s_i^2 = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2/(n_i - 1)$$

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Estimation of Common mean

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- $$\hat{\mu}^{GD} = \frac{n_1 s_2^2}{n_1 s_2^2 + n_2 s_1^2}\bar{X}_1 + \frac{n_2 s_1^2}{n_1 s_2^2 + n_2 s_1^2}\bar{X}_2$$

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- and gave a necessary and sufficient condition on n_1 and n_2 for $\hat{\mu}^{GD}$ to have a smaller variance than both $\bar{X}_1$ and $\bar{X}_2$.

Estimation of Common mean when $\sigma_1^2 \leq \sigma_2^2$

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$$\hat{\mu}^{Nair} = \begin{cases} \hat{\mu}^{GD}, & if\ s_1^2 \leq s_2^2 \\ \frac{n_1}{n_1+n_2}\bar{X}_1 + \frac{n_2}{n_1+n_2}\bar{X}_2, & if\ s_1^2 > s_2^2, \end{cases}$$

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- showed that $\hat{\mu}^{Nair}$ has smaller variance than $\hat{\mu}^{GD}$.

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- Elfessi and Pal (1992) showed that $\hat{\mu}^{Nair}$ stochastically dominates $\hat{\mu}^{GD}$. (As for the definitions of stochastic dominance and universal dominance, see Hwang (1985).)

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- √ Lee (1981) showed that RMLE (restricted MLE) uniformly improves upon sample means under MSE.
- √ Kelly (1989) and Hwang and Peddada (1994) proved that RMLE universally dominates sample means.

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- Oono and Shinozaki (2005) proposed a truncated estimators of $\mu_i, i = 1, 2$,

$$\hat{\mu}_1^{OS} = \min\{\bar{X}_1, \hat{\mu}^{GD}\}, \quad \hat{\mu}_2^{OS} = \max\{\bar{X}_2, \hat{\mu}^{GD}\},$$

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- they showed that $\hat{\mu}_i^{OS}$ uniformly improves upon $\bar{X}_i$, if and only if the risk of $\hat{\mu}_i^{OS}$ is not larger than that of $\bar{X}_i$ when $\mu_1 = \mu_2$. (See also Garren (2000).)

When order restrictions are given on both means and variances

- ✓ When order restrictions are given on both means and variances, Chang and Shinozaki (2010) have considered the estimators based on the estimators given by Oono and Shinozaki (2005) as follow:

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$$\hat{\mu}_1^{CS} = \begin{cases} \hat{\mu}_1^{OS}, & if\ s_1^2 \le s_2^2 \\ \min\{\bar{X}_1, \frac{n_1}{n_1+n_2}\bar{X}_1 + \frac{n_2}{n_1+n_2}\bar{X}_2\}, & if\ s_1^2 > s_2^2. \end{cases}$$

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- $$\hat{\mu}_2^{CS} = \begin{cases} \hat{\mu}_2^{OS}, & if\ s_1^2 \leq s_2^2 \\ \max\{\bar{X}_2, \frac{n_1}{n_1+n_2}\bar{X}_1 + \frac{n_2}{n_1+n_2}\bar{X}_2\}, & if\ s_1^2 > s_2^2. \end{cases}$$

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- ✓ They also showed that $(\hat{\mu}_1^{CS}, \hat{\mu}_2^{CS})$ stochastic dominates $(\hat{\mu}_1^{OS}, \hat{\mu}_2^{OS})$ when estimating (μ_1, μ_2), simultaneously.

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- ✓ They also showed that $(\hat{\mu}_1^{CS}, \hat{\mu}_2^{CS})$ stochastic dominates $(\hat{\mu}_1^{OS}, \hat{\mu}_2^{OS})$ when estimating (μ_1, μ_2), simultaneously.
- ✓ Shi (1994) and Ma and Shi (2002) discussed the order restricted MLE of μ_i and σ_i^2 under squared error loss.

A wider class estimators of two means

- Let that $\gamma, \tilde{\gamma}$, and γ^+ are functions of n_1, n_2, s_1^2, s_2^2, and $\bar{X}_1 - \bar{X}_2$ and that $0 \leq \gamma, \tilde{\gamma}, \gamma^+ \leq 1$ and,

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- $$\gamma^+ = \begin{cases} \gamma, & if\ \gamma \geq \frac{n_1}{n_1+n_2}, \\ \tilde{\gamma}, & if\ \gamma < \frac{n_1}{n_1+n_2}. \end{cases}$$

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- ✓ Chang, Oono and Shinozaki(2011C) have considered estimators of the forms

$$\hat{\mu}_1(\gamma) = \min\{\bar{X}_1, \gamma\bar{X}_1 + (1-\gamma)\bar{X}_2\} \qquad (1.4)$$

and

$$\hat{\mu}_2(\gamma) = \max\{\bar{X}_2, \gamma\bar{X}_1 + (1-\gamma)\bar{X}_2\}. \qquad (1.5)$$

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- if we choose $\tilde{\gamma}$ so that

$$\frac{n_1}{n_1+n_2} \le \tilde{\gamma} \le 2\frac{n_1}{n_1+n_2} - \gamma, \quad \text{when } \gamma < \frac{n_1}{n_1+n_2}. \tag{3.2}$$

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- ✓ $\hat{\mu}_1(\gamma)$ has smaller MSE than $\hat{\mu}_1(\gamma^+)$ for sufficiently large $\Delta = \mu_2 - \mu_1$.

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- then $(\hat{\mu}_1(\gamma^+), \hat{\mu}_2(\gamma^+))$ dominates $(\hat{\mu}_1(\gamma), \hat{\mu}_2(\gamma))$ in the sense that
- $$P\left\{\sum_{i=1}^{2}\left(\frac{\hat{\mu}_i(\gamma^+) - \mu_i}{\tau_i}\right)^2 \le d\right\} \ge P\left\{\sum_{i=1}^{2}\left(\frac{\hat{\mu}_i(\gamma) - \mu_i}{\tau_i}\right)^2 \le d\right\}$$

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- For the case, when the estimators are equal with positive probability, Nayak (1990) modified Pitman's criterion as follows :
- T_1 is said to be closer to θ than T_2 if

$$P_r\{|T_1 - \theta| < |T_2 - \theta|\} > \frac{1}{2} P_r\{T_1 \neq T_2\}.$$

Modified Pitman's criterion

- Motived by Nayak (1990), Gupta and Sinha (1992) defined the modified Pitman nearness (MPN) of T_1 compared to T_2. Setting

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$$MPN_\theta(T_1, T_2) = P_r\{|T_1 - \theta| < |T_2 - \theta| | T_1 \neq T_2\}$$
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are closer to respective means than $\bar{X}_i, i = 1, 2,$ that is, $MPN_{\mu_i}(\hat{\mu}_i^{GS}, \bar{X}_i) > 1/2,\ i = 1, 2,$.

Our results under modified Pitman's criterion

- ✓ **Theorem**
  Suppose that $P\{\gamma < n_1/(n_1 + n_2)\} > 0$, then the estimator $\hat{\mu}_2(\gamma^+)$ is closer to μ_2 than $\hat{\mu}_2(\gamma)$,

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✓ **Theorem**
The estimator $\hat{\mu}_1(\gamma^+)$ is not closer to μ_1 than $\hat{\mu}_1(\gamma)$ for sufficiently large Δ.

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$$\begin{aligned} & MPN_{\boldsymbol{\mu}}(\hat{\boldsymbol{\mu}}(\gamma^+), \hat{\boldsymbol{\mu}}(\gamma)) \\ = & \frac{P_r\{\sum_{i=1}^{2}(\hat{\mu}_i(\gamma^+) - \mu_i)^2/\tau_i^2 \leq \sum_{i=1}^{2}(\hat{\mu}_i(\gamma) - \mu_i)^2/\tau_i^2, \hat{\boldsymbol{\mu}}(\gamma^+) \neq \hat{\boldsymbol{\mu}}(\gamma)\}}{P_r\{\hat{\boldsymbol{\mu}}(\gamma^+) \neq \hat{\boldsymbol{\mu}}(\gamma)\}} \\ > & 1/2. \end{aligned}$$

Conclusion

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- √ Rao (1981) compared the minimum MSE and Pitman's closeness criteria and suggested that Pitman's closeness criterion could be used as an alternative criterion to compare estimators.
- √ However, Blyth (1972) pointed out the intransitivity drawback of Pitman's closeness criterion and also pointed out that there is some inconsistency among Pitman's closeness, minimum MSE, and minimum mean absolute error creiteria.

Conclusion

- √ For the estimation problem of two ordered normal means with ordered variances, we have confirmed that the result obtained by using the Pitman's closeness criterion is consistent with the one obtained by using the MSE criterion.

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- √ For the estimation problem of two ordered normal means with ordered variances, we have confirmed that the result obtained by using the Pitman's closeness criterion is consistent with the one obtained by using the MSE criterion.
- √ Thank you for your attention.