On the pricing of Investment Corporation Bonds

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Main Results

- We will propose a method to estimate recovery rates and default probabilities of investment corporation bonds using corporate bond pricing model proposed by Kariya and Tsuda (1994) and Tsuda (2003).

- Characteristics of our corporate bond pricing model

1. Estimating recovery rates using financial statements and of investment corporation that issues bond.
2. Default probability depending on the attributes of each bond
3. Stochastic discount rate
4. Empirical model

- Empirical results show that our method performs well in predicting credit risk changes.

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Introduction

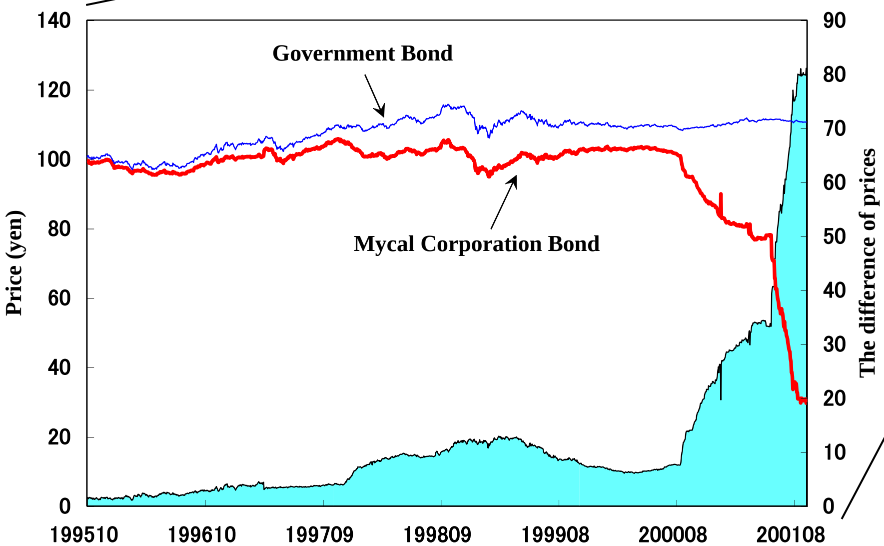
Our purpose of corporate bond price model

- Estimation of the reasonable prices of corporate bond
- Prediction of rating changes
- Credit risk management of portfolio

Information

① Default probability
② Recovery rate
③ Expected loss
④ The prediction of rating changes

Data



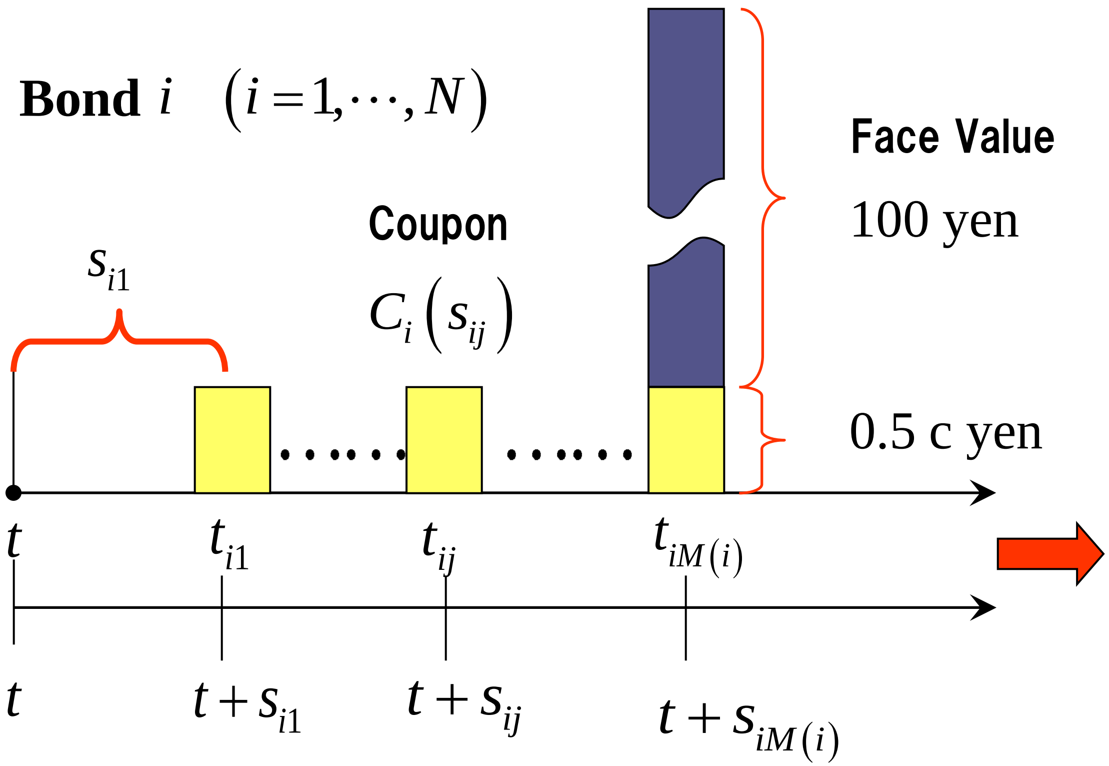
Bond pricing model

Previous Studies of Corporate Bond Pricing Model

- Kariya (1993) : Cross-sectional bond pricing model for individual bonds
- Kariya and Tsuda (1994) : Generalized the cross-sectional model to time-dependent Markov models
- Tsuda (2003) : Dynamic bond pricing model

Generation Time and Term of the Cash Flows

Bond $i \quad (i = 1, \cdots, N)$



The generation time of the cash flows of Bond i

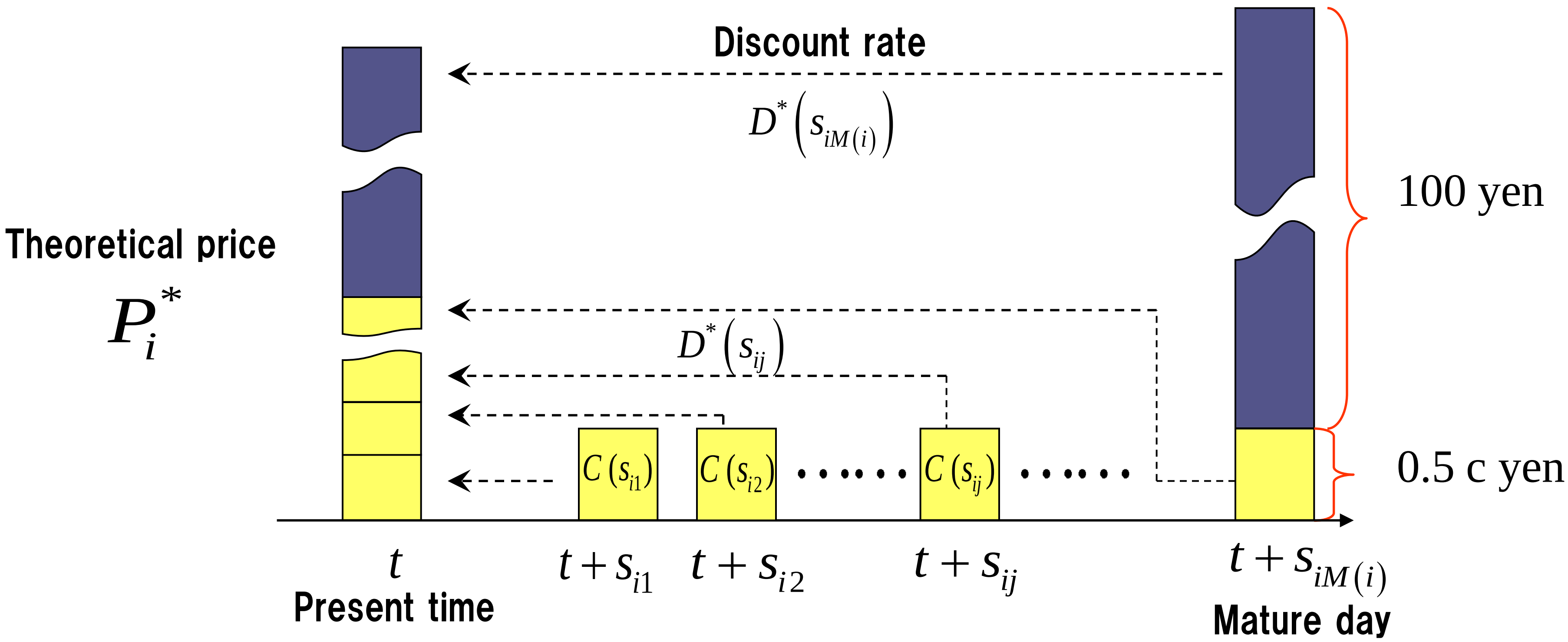
$$s_{ij} = t_{ij} - t \quad (j = 0, \cdots, M(i)), \quad t_{i0} = t, \quad s_{i0} = 0$$

$$C_i(s_{i1}) = C_i(s_{i2}) = \cdots = C_i(s_{iM(i)-1}) = 0.5c, \quad C_i(s_{iM(i)}) = 100 + 0.5c$$

Theoretical bond price

- Generally the theoretical bond price is basically the total sum of cash flows $C(s_j)$ in period s_j discounted by the corresponding discount rate $D^*(s_j)$ and it is called the present value.

$$P_i^* = C(s_{i1})D^*(s_{i1}) + C(s_{i2})D^*(s_{i2}) + \cdots + C(s_{iM(i)})D^*(s_{iM(i)}) = \sum_{j=1}^{M(i)} C(s_{ij})D^*(s_{ij})$$

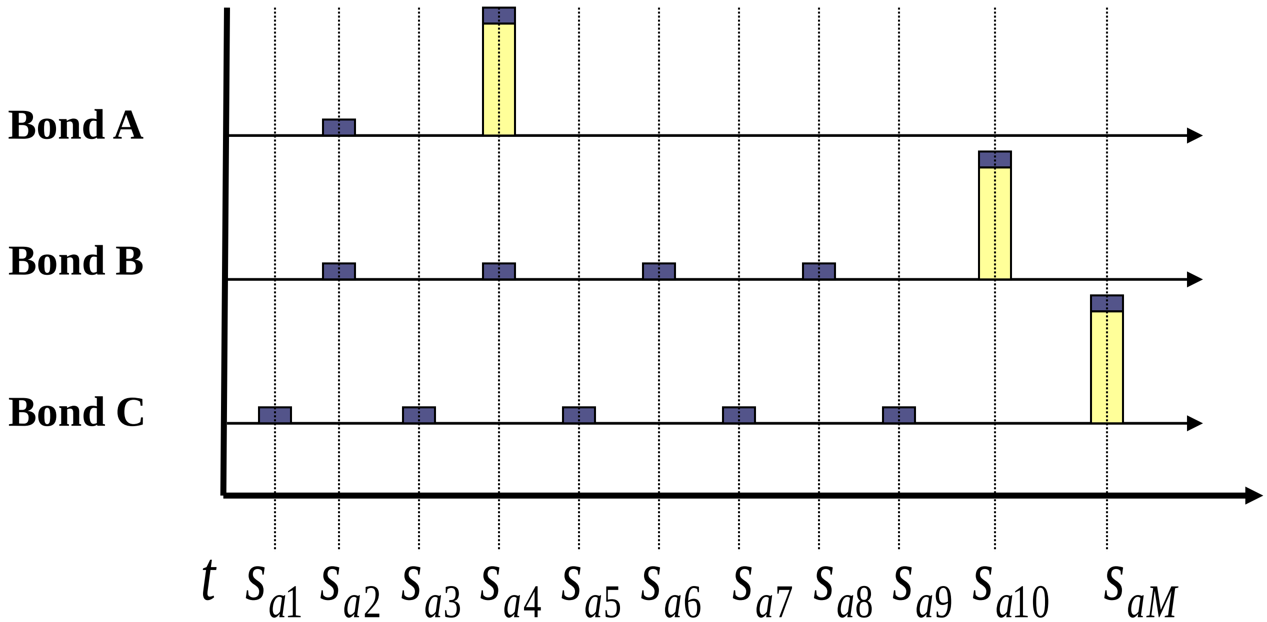


Our Approach

- The *j*-th cash flow of the *i*-th bond occurring at $t+s_j$ are represented by $C_{it}(s_j)$.
- All these terms in N bonds are enumerated in an increasing order as

$$s_{a1} < s_{a2} < \cdots < s_{aM}, \quad s_{aM} = \max\{s_{1M(1)}, \cdots, s_{NM(N)}\}$$

- Let $C_i(s)$ be the cash flow function of the *i*-th bond defined on $0 < s < s_{aM}$.



Our Bond Pricing Model

Bond Price (in case of no credit risk)

$$P_i(0) = \sum_{m=1}^{M} C_i(s_{am}) D(s_{am})$$

When $s_{am} \neq s_{ij}$, $C_i(s_{am}) = 0$.

Stochastic Discount Rate

$$D(s_{am}) = \underbrace{\overline{D}(s_{am})}_{\text{Mean discount rate}} + \underbrace{(D(s_{am}) - \overline{D}(s_{am}))}_{\text{Random discount factor}} = \overline{D}(s_{am}) + \Delta(s_{am})$$

Stochastic Process of Discount Rates

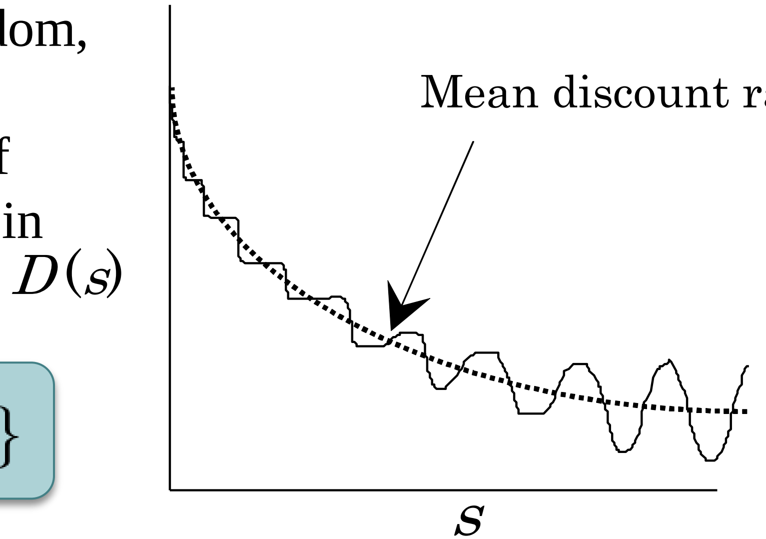
Bond Price

$$P_i(0) = \boldsymbol{C}_i'\boldsymbol{D}$$
$$= \boldsymbol{C}_i'(\overline{\boldsymbol{D}} + D) = \boldsymbol{C}_i'\overline{\boldsymbol{D}} + u_i$$

$$\overline{\boldsymbol{D}} = E(\boldsymbol{D}),\ D = \boldsymbol{D} - \overline{\boldsymbol{D}},\ u_i = \boldsymbol{C}_i'D$$
$$\boldsymbol{C}_i = (C_i(s_{a1}), \cdots, C_i(s_{aM}))',\ \boldsymbol{D} = (D(s_{a1}), \cdots, D(s_{aM}))'$$

The realization of price ,which is random, is viewed as equivalent to a whole realization of the stochastic process of the discount function of the *i*-th bond in the model.

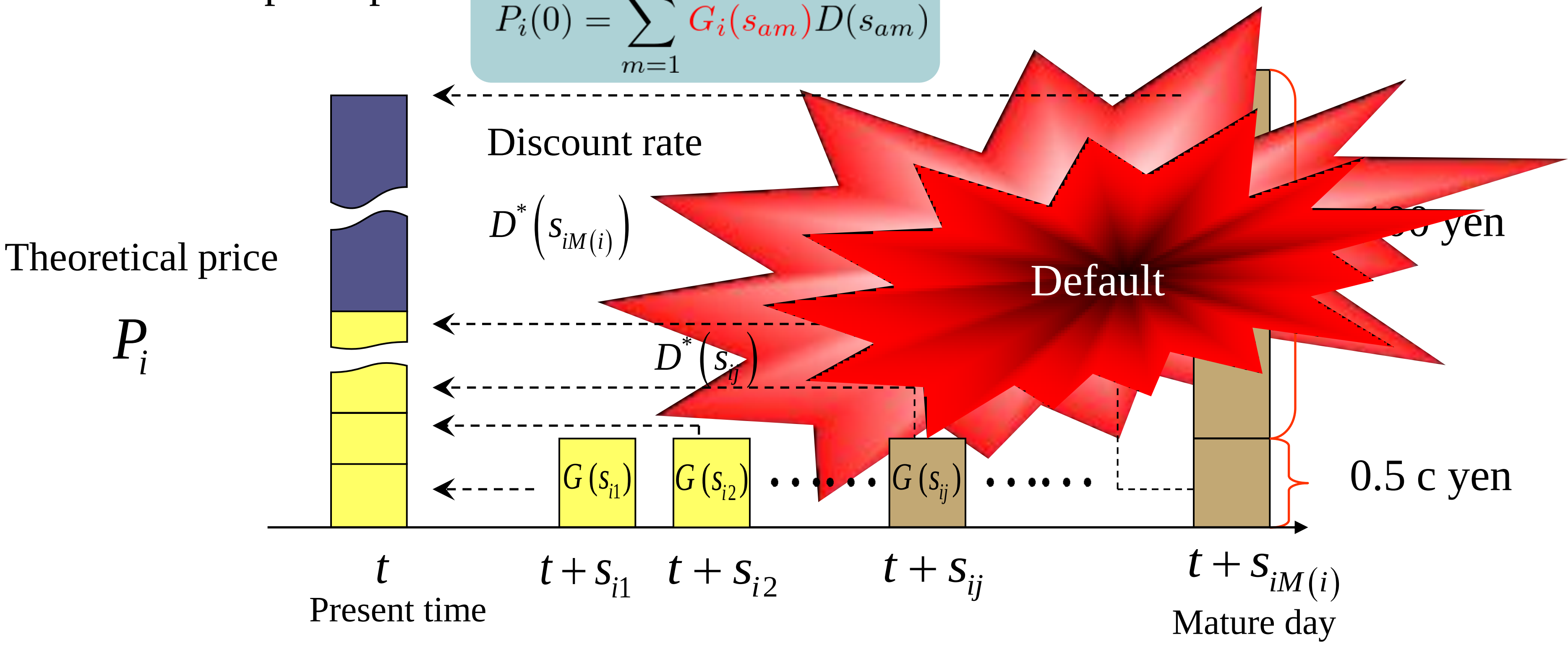
$$\boldsymbol{D} = \{D(s)\ :\ 0 \leq s \leq s_{aM}\}$$



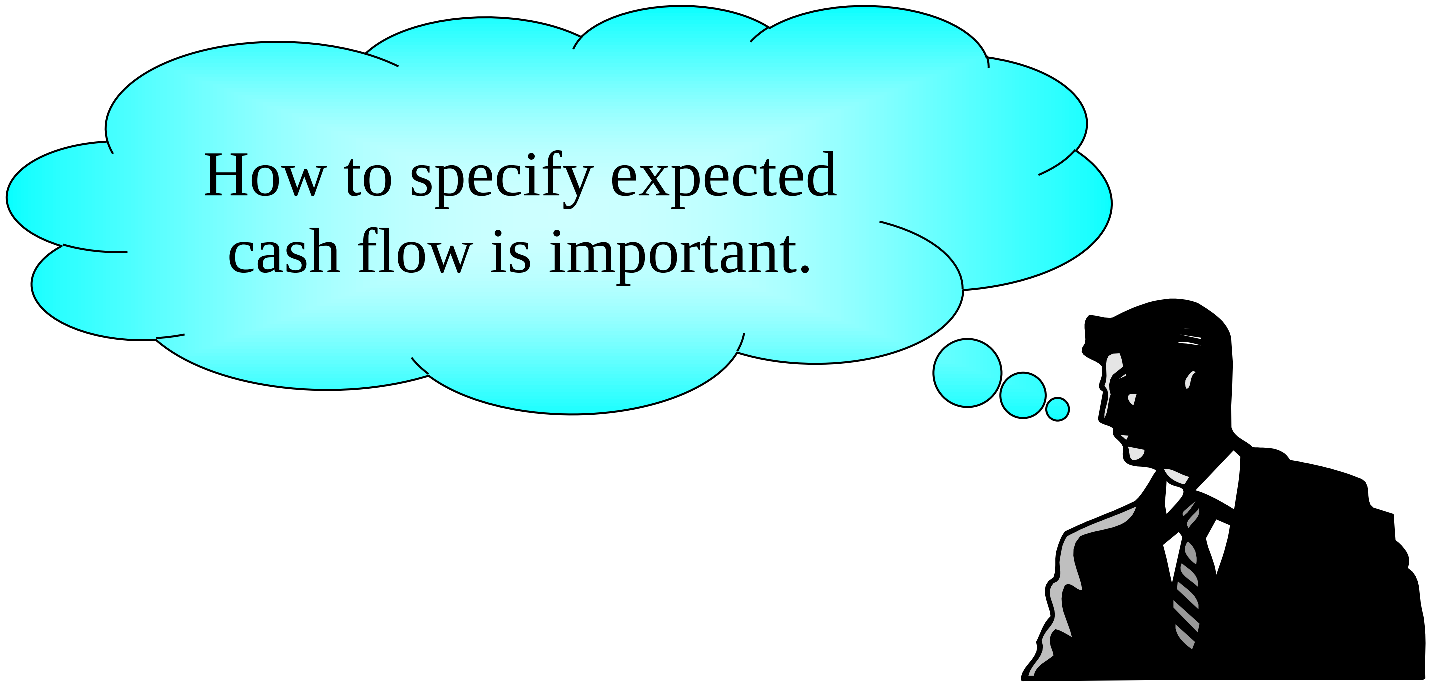
Corporate Bond Price

- The characteristic of corporate bond has default possibility so-called credit risk comparing with ordinary government bond.
- Therefore the cash flows $G_i(s_{am})$ in future periods of corporate bonds are stochastic.
- The market prices of corporate bond are reflected in arrearages or default of interest and principal.

$$P_i(0) = \sum_{m=1}^{M} G_i(s_{am}) D(s_{am})$$



Default Probability

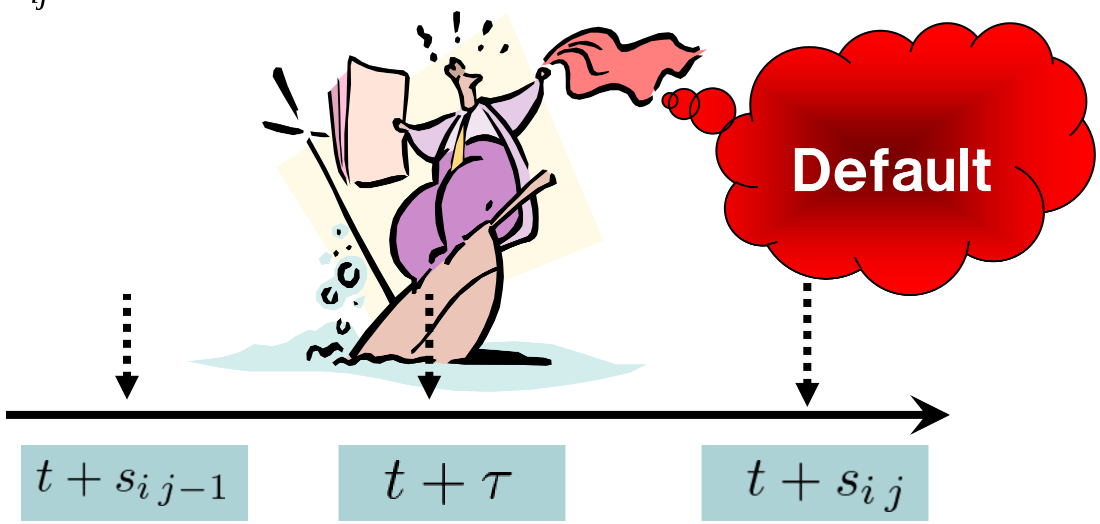


- We assume that the default probability $h_i\left(s_{ij}\right)$ until $t+s_{ij}$ time of the i-th corporate bond depends on the bond rating k.

$$h_i\left(s_{ij};k\right) \quad \left(k=1,\cdots,K\right)$$

Expected Cash Flow

- We assume that the principal and the interest of a corporate bond are recovered at $t+s_{ij}$ time when default occurs at time $t+\tau$ between $t+s_{i\,j-1}$ time and $t+s_{ij}$ time.



- The expected cash flow at $t+s_{ij}$ time of the *i*-th corporate bond of credit rating k

$$E\left(C_i^*\left(s_{i\,j}\right)\right)=C_i\left(s_{i\,j}\right)\times\underbrace{\left[1-h_i\left(s_{i\,j}\right)\right]}_{\text{Survival probability}}+0\times\underbrace{h_i\left(s_{i\,j}\right)}_{\text{Default probability}}\quad\cdots\cdots\cdots\quad(A)$$

Expected Cash Flow

The expected cash flow of the recovered value

$$E\left(C_i^{**}\left(s_{i\,j}\right)\right)=C_i\left(s_{i\,M(i)}\right)\overbrace{\gamma\left(k(i)\right)}^{\text{Recovery rate}}\times\underbrace{\left\{h_i\left(s_{i\,j}\right)-h\left(s_{i\,j-1}\right)\right\}}_{\text{Default probability of the period between }(s_{ij-1},\,s_{ij})}+0\times\left[\overbrace{1-\left\{h_i\left(s_{i\,j}\right)-h\left(s_{i\,j-1}\right)\right\}}^{\text{Survival probability of the period between }(s_{ij-1},\,s_{ij})}\right]\quad\cdots\cdots\quad(B)$$

- We assume that the recovery rate $\gamma(k(i))$ depends only corporate rating.

Expected Cash Flow

Expected cash flow : (A) + (B)

$$
\begin{aligned}
E(G_i(s_{ij})) &= E[C_i(s_{ij})1_{\{s_{ij}<\tau\}} + 100\gamma(k(i))1_{\{s_{i,j-1}<\tau<s_{ij}\}}] \\
&= C_i(s_{ij})[1-h_i(s_{ij})] + C_i(s_{i\,M(i)})\gamma(k(i))[h_i(s_{ij}) - h_i(s_{i,j-1})] \\
&= C_i(s_{ij}) - \underbrace{\{C_i(s_{ij})h_i(s_{ij}) - C_i(s_{i\,M(i)})\gamma(k(i))[h_i(s_{ij}) - h_i(s_{i,j-1})]\}}_{E(L_i(s_{ij}))}
\end{aligned}
$$

$$E(G_i(s_{ij})) = C_i(s_{ij}) - E(L_i(s_{ij})) = \overline{G}_i(s_{ij})$$

$$E(L_i(s_{ij})) = C_i(s_{ij})h_i(s_{ij}) - C_i(s_{iM(i)})\gamma(k(i))[h_i(s_{ij}) - h_i(s_{i,j-1})] = \overline{L}_i(s_{ij})$$

Corporate Bond Pricing Model

Cash flow

$$\begin{aligned}
\boldsymbol{G}_i &= \overline{\boldsymbol{G}}_i + (\boldsymbol{G}_i - \overline{\boldsymbol{G}}_i) \\
&= (\boldsymbol{C}_i - \overline{\boldsymbol{L}}_i) + \{(\boldsymbol{C}_i - \boldsymbol{L}_i) - (\boldsymbol{C}_i - \overline{\boldsymbol{L}}_i)\} \\
&= (\boldsymbol{C}_i - \overline{\boldsymbol{L}}_i) - (\boldsymbol{L}_i - \overline{\boldsymbol{L}}_i)
\end{aligned}$$

Let $G_i(s)$ be the cash flow function of the *i*-th bond defined on $0 < s < s_{aM}$

Bond price

Expected loss

Mean discount rate

$$\begin{aligned}
P_i(0) &= \sum_{m=1}^{M} G_i(s_{am}) D(s_{am}) \\
&= \boldsymbol{G}_i' \boldsymbol{D} \\
&= (\boldsymbol{C}_i - \overline{\boldsymbol{L}}_i)' \boldsymbol{D} + \nu_i \\
&= (\boldsymbol{C}_i - \overline{\boldsymbol{L}}_i)' (\overline{\boldsymbol{D}} + \boldsymbol{\Delta}) + \nu_i \\
&= (\boldsymbol{C}_i - \overline{\boldsymbol{L}}_i)' \overline{\boldsymbol{D}} + \varepsilon_i
\end{aligned}$$

Mean Discount Function

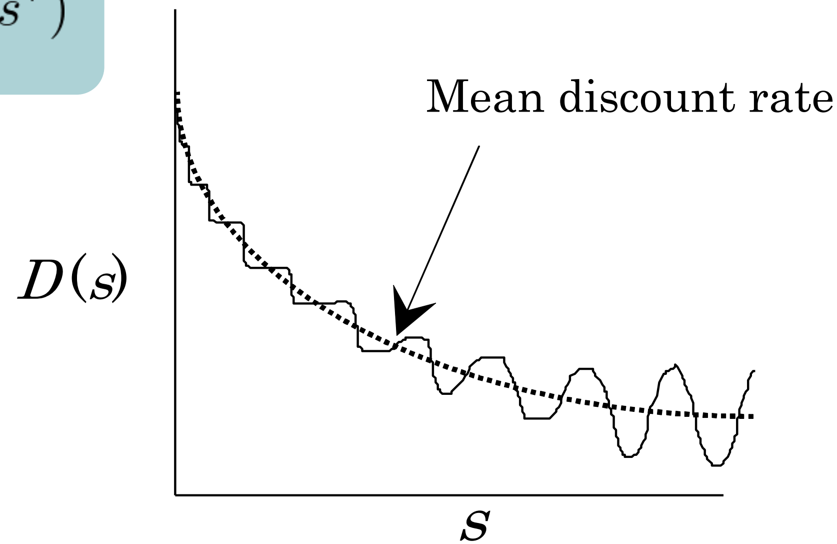
The mean discount function is specified as exponential function

$$\overline{D}(s) = \exp(-\kappa s)$$

$$s = -\frac{1}{\alpha}\log(1-s^*), \quad 0 \leq s^* < 1$$

$$\overline{D}(s) = \overline{D}\left[-\frac{1}{\alpha}\log(1-s^*)\right] \equiv \overline{\Theta}(s^*)$$

$$\overline{\Theta}(s^*) = (1-s^*)^{\frac{\kappa}{\alpha}}$$



Polynomial for mean discount function

$$\overline{\Theta}(s^*) = 1 + \sum_{j=1}^{p} \delta_j s^{*j}$$

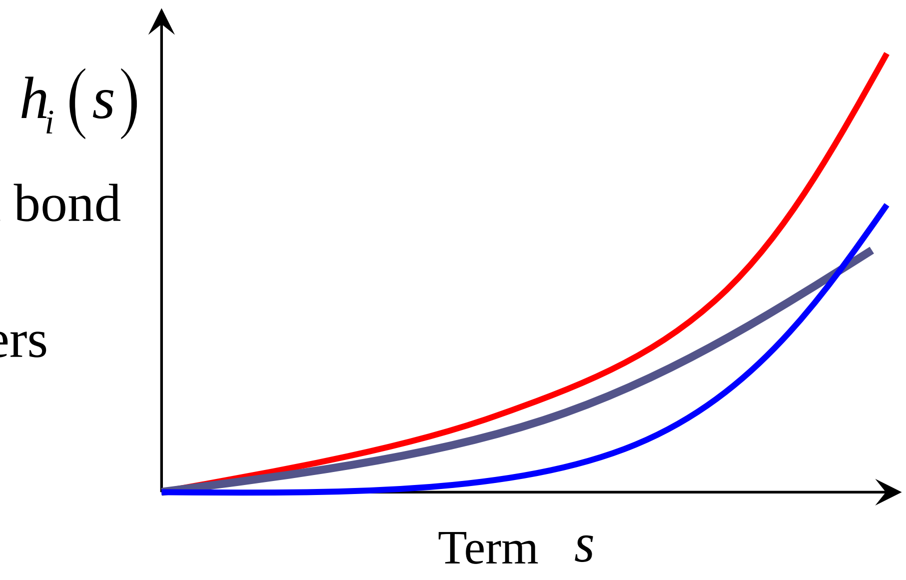
p : degree of polynomial function

Cumulative Default Probability Model

The cumulative default probability h_i until $t+s_{ij}$ time from t time of the company

Polynomial

$$
\begin{aligned}
h_i(s) &= \zeta_1(\boldsymbol{z}_i)s + \cdots + \zeta_p(\boldsymbol{z}_i)s^p \qquad (i = 1, \cdots, N), \\
\zeta_j(\boldsymbol{z}_i) &= \zeta_{l1}z_{i1} + \cdots + \zeta_{lq}z_{iq} \qquad (l = 1, \cdots, q).
\end{aligned}
$$



z_{ij} : the attribute variables of the *i*-th bond

$\zeta_{i1}, \zeta_{i2}, \cdots, \zeta_{iq}$: the common parameters among N bonds

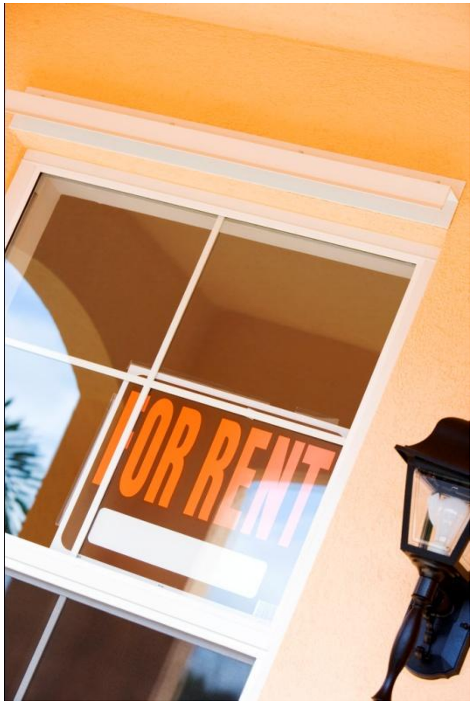
Empirical Analysis

- Prediction of Investment Corporation Bonds Prices

- 13 investment corporation (51 brand)
- Japanese Government bonds (JGBs)　　Bloomberg.L.P.
  - Maturity of 10 years,　Face value of 100 yen
- Market data of Investment corporation bonds
- Financial statement of investment corporation
- Bond Rating Agency
  – Rating & Investment Information,Inc. (R&I)
  – Japan Credit Rating Agency, Ltd.(JCR)
  – Standard & Poor's (S&P)

Investment corporation

- REITs（Real Estate Investment Trusts）
  - Investing in REITs is indirect real estate investment.
  - REITs may not have to pay corporate taxes.
  - The investment management of the real estate is then entrusted to third party asset management companies, that is, special purpose company (SPC)
  - In Japan, the owner of the real estate is called an investment corporation.



J-REIT

- 2001/9 J-REIT market open. (2 brand)
- 2007 41 brand (Tokyo Stock Exchange,Inc)
- 2007/9 The Lehman Brothers Holdings Co. failed.
- 2008/10 New City Residence Investment Corp failed.

Investment Corporation Bonds

Investment corporation (Maturity)	Amount of issue (billion yen)	Coupon rate	Issue Date	Redemption Date
Global One Real Estate (5)	250	1.08	2005/10/21	2010/10/21
Global One Real Estate (7)	100	1.51	2005/10/21	2012/10/19
Kenedix Realty (5)	90	1.74	2007/3/15	2012/3/15
Kenedix Rality (10)	30	2.37	2007/3/15	2017/3/15
Japan Real Estate (7)	100	0.98	2003/4/30	2010/4/30
Japan Real Estate (20)	100	2.56	2005/9/29	2025/9/29
Japan Real Estate (5)	100	1.67	2007/6/18	2012/6/18
Japan Real Estate (7)	150	1.91	2007/6/18	2014/6/18
TOKYU REIT (5)	50	1.65	2007/10/24	2012/10/24

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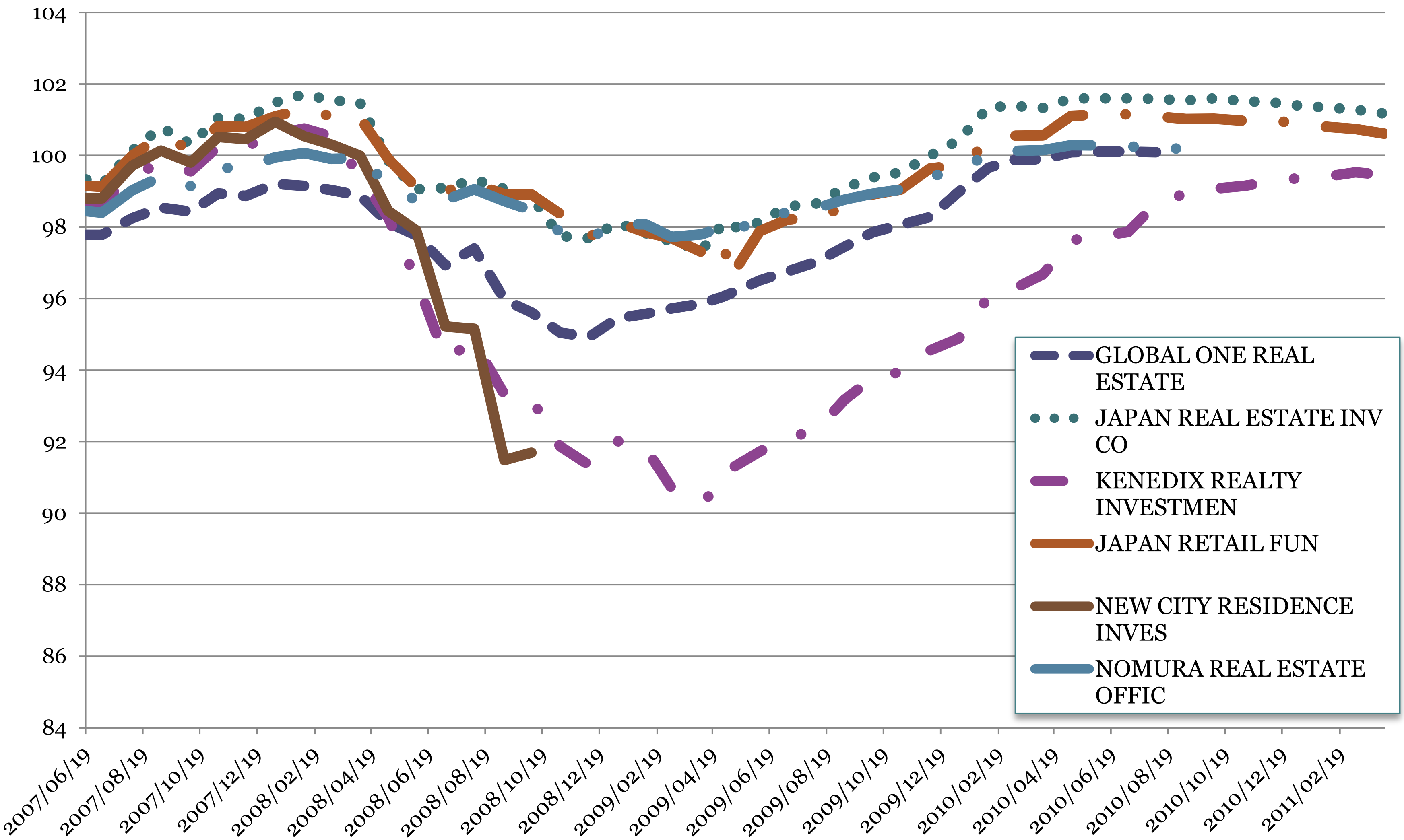
Market Price of Investment Corporation Bonds (2007/6ー2011/4)



Estimation of Mean Discount Function

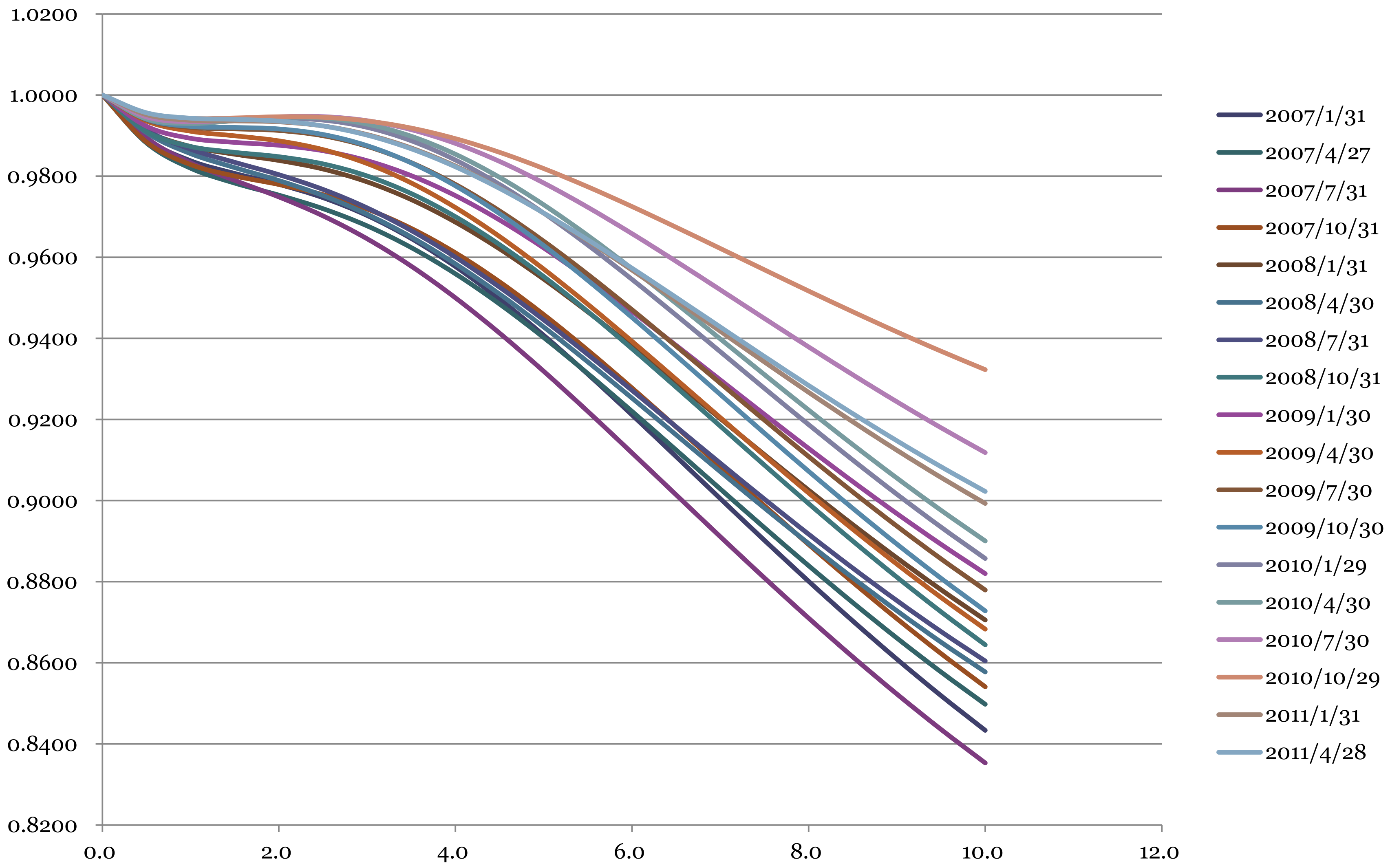
$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\eta}$$

$$\begin{aligned}
\boldsymbol{y} &= (y_1, \cdots, y_N)', \qquad y_i = P_i(0) - \sum_{m=1}^{M} C_i(s_{am}) \\
\boldsymbol{\beta} &= (\delta_1, \cdots, \delta_p)', \\
\boldsymbol{X} &= (\boldsymbol{x}_1, \cdots, \boldsymbol{x}_N)', \qquad \boldsymbol{x}_i = (x_{i1}, \cdots, x_{ip})', \\
x_{ir} &= \sum_{m=1}^{M} s_{am}^{*r} C_i(s_{am}^{*}), \quad \boldsymbol{\eta} = (\eta_1, \cdots, \eta_N)'
\end{aligned}$$

$$\overline{\Theta}_t(s_t^*) = 1 + \delta_1 s_t^* + \delta_2 s_t^{*2} + \delta_3 s_t^{*3}$$

	δ1	δ2	δ3
2007/1/31	−0. 149	0. 426	−0. 536
2007/4/27	−0. 160	0. 429	−0. 514
2007/7/31	−0. 140	0. 347	−0. 470
2007/10/31	−0. 161	0. 469	−0. 552
2008/1/31	−0. 125	0. 390	−0. 484
2008/4/30	−0. 125	0. 326	−0. 431
2008/7/31	−0. 116	0. 309	−0. 418
2008/10/31	−0. 131	0. 436	−0. 538
2009/1/30	−0. 114	0. 389	−0. 481
2009/4/30	−0. 096	0. 341	−0. 470

Mean Discount Function



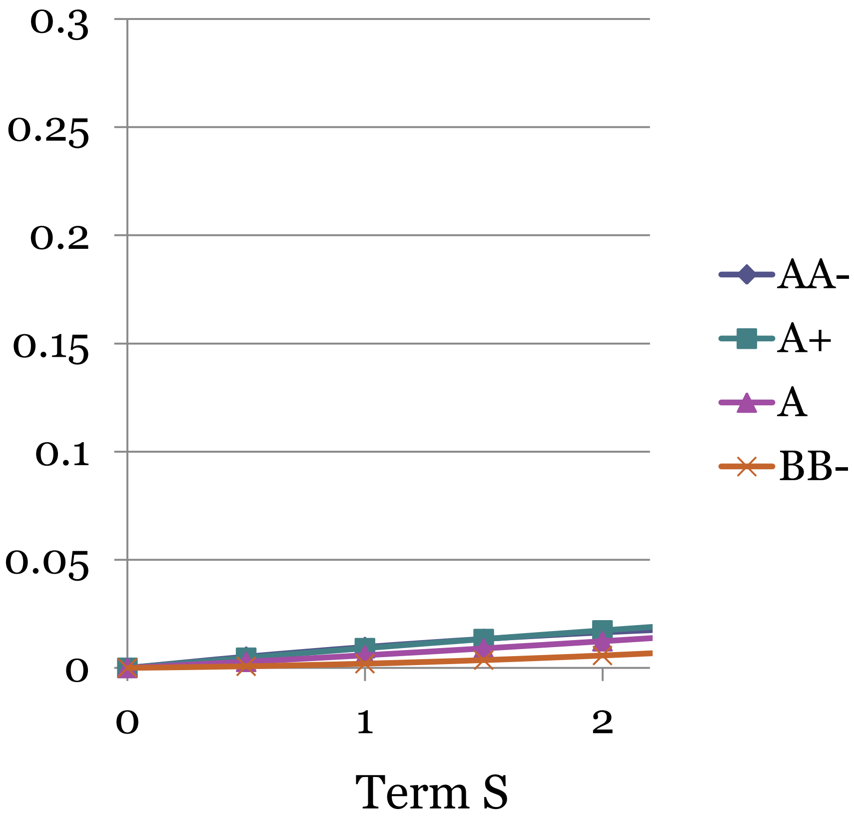
Estimation of Default Probability Function

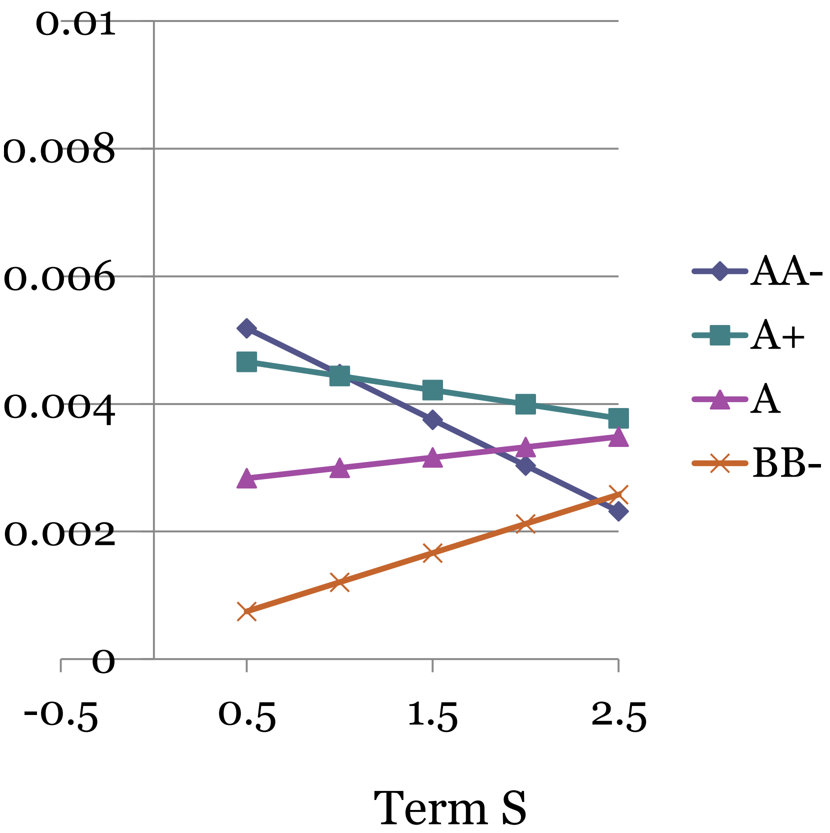
- Linear Model

$$\boldsymbol{y}^* = \boldsymbol{X}^*\boldsymbol{\beta}^* + \boldsymbol{\varepsilon}^*$$

$$\begin{aligned}
\boldsymbol{y}^* &= (y_1^*, \cdots, y_N^*)', \quad y_i^* = P_i(0) - \sum_{m=1}^{M} C_i(s_{am})\overline{D}(s_{am}), \\
\boldsymbol{\beta}^* &= (\boldsymbol{\zeta}_1', \cdots, \boldsymbol{\zeta}_p')', \quad \boldsymbol{\zeta}_\ell = (\zeta_{\ell 1}, \cdots, \zeta_{\ell q})' \\
\boldsymbol{X}^* &= (\boldsymbol{x}_1^*, \cdots, \boldsymbol{x}_N^*)', \quad \boldsymbol{x}_i^* = (\boldsymbol{x}_{i1}^*, \cdots, \boldsymbol{x}_{ip}^*)', \quad \boldsymbol{x}_{ir}^* = (x_{i1r}^*, \cdots, x_{iqr}^*)', \\
x_{i\ell r}^* &= -\sum_{m=1}^{M} z_{iv}[C_i(s_{am})s_{am}^r - 100\gamma(k(i))\{s_{am}^r - s_{am-1}^r\}]\overline{D}(s_{am}), \\
\boldsymbol{\varepsilon} &= (\varepsilon_1, \cdots, \varepsilon_N)'
\end{aligned}$$

Cumulative Default Probability Model and Hazard Rate





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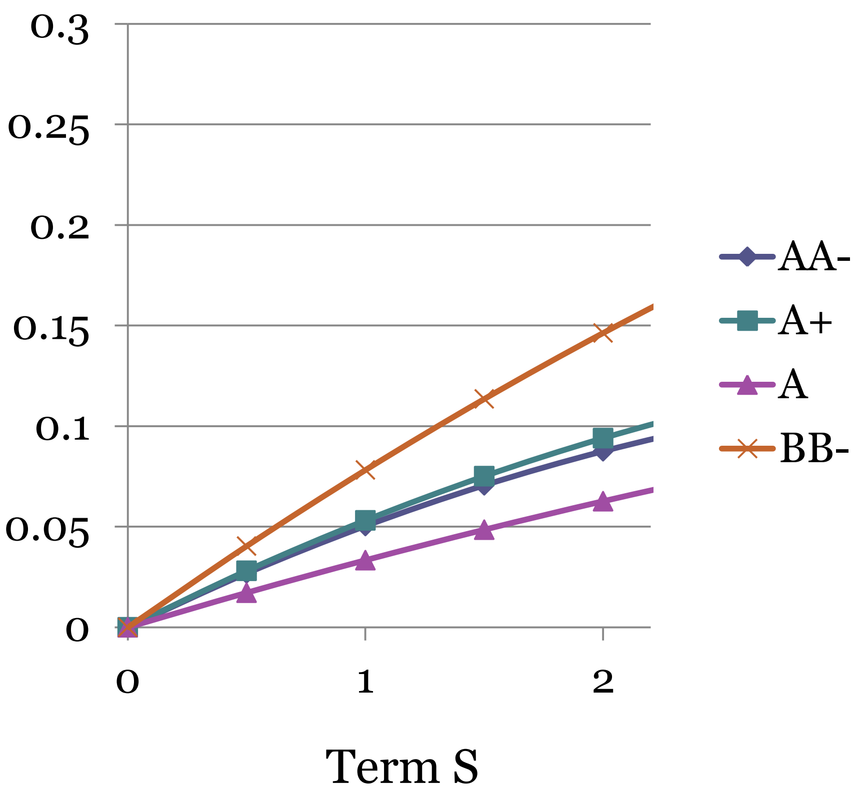
Cumulative Default Probability Model and Hazard Rate



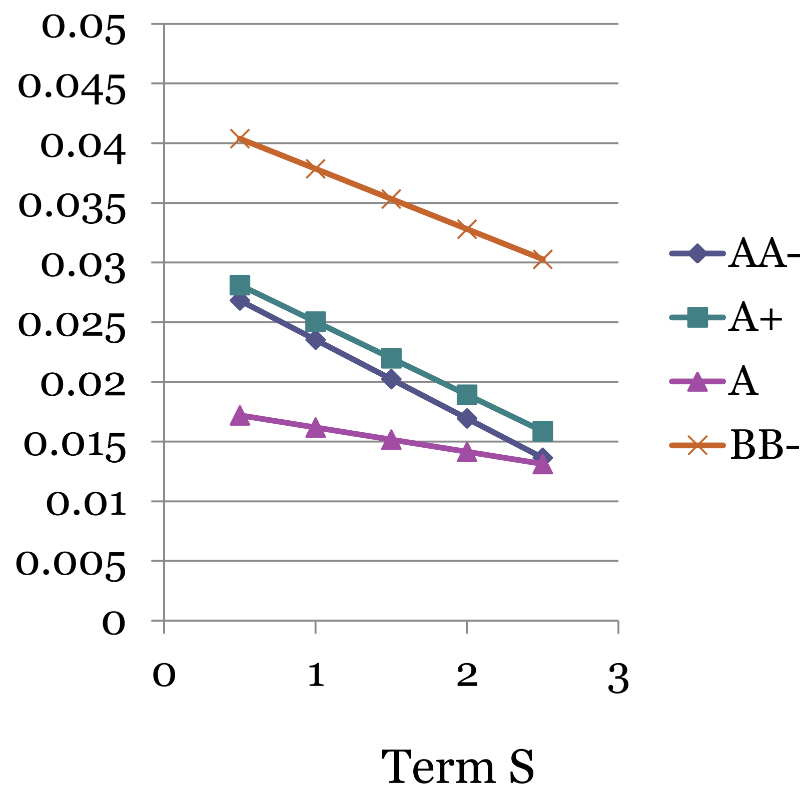


Table 1：Estimates of the default probability function

期間	ζ_1				ζ_2			
	AA−	A+	A	BB−	AA−	A+	A	BB−
2007/4/27	0.0354	0.0464	0.0506	0.1934	0.0018	0.0026	0.0003	−0.0228
	3.123	2.848	7.156	7.143	2.086	1.298	0.586	−3.753
2007/7/31	0.0382	0.0517	0.0524	0.2083	0.0017	0.0021	0.0002	−0.0260
	3.107	3.005	6.945	7.306	1.835	1.005	0.384	−3.893
2007/10/31	0.0816	0.1017	0.0977	0.4458	0.0002	0.0010	−0.0014	−0.0612
	3.443	3.029	6.954	6.334	0.127	0.242	−1.419	−3.508
2008/1/31	0.0879	0.0917	0.0854	0.4318	−0.0006	0.0009	−0.0010	−0.0630
	3.361	2.813	6.300	6.203	−0.298	0.208	−1.055	−3.472
2008/4/30	0.0950	0.0962	0.0909	0.4960	−0.0010	0.0012	−0.0012	−0.0784
	3.331	2.635	6.064	6.028	−0.475	0.238	−1.171	−3.467
2008/7/31	0.0698	0.0791	0.0690	0.4168	0.0004	0.0009	−0.0004	−0.0705
	2.771	2.538	5.414	6.167	0.211	0.214	−0.481	−3.604
2008/10/31	0.1491	0.1688	0.1407	0.9178	−0.0036	−0.0032	−0.0035	−0.1783
	3.206	2.845	5.985	6.200	−0.978	−0.371	−2.055	−3.874
2009/1/30	0.1151	0.1680	0.1393	1.0256	−0.0014	−0.0031	−0.0033	−0.2169
	2.670	2.687	5.580	6.143	−0.413	−0.335	−1.844	−3.931
2009/4/30	0.1058	0.1550	0.1251	1.1268	−0.0009	−0.0016	−0.0025	−0.2603
	2.179	2.210	4.463	5.905	−0.247	−0.157	−1.227	−3.891

Upper：results, Lower：t value

Table 2：Standard Deviation of estimation errors

	AA−	A+	A	BB−
2007/4/27	2.895	4.559	3.186	7.517
2007/7/31	2.811	4.617	2.827	7.790
2007/10/31	2.481	4.159	2.211	10.440
2008/1/31	2.325	5.237	2.243	10.514
2008/4/30	2.121	4.795	2.039	11.433
2008/7/31	2.446	5.215	2.380	11.230
2008/10/31	2.141	4.558	1.003	12.122
2009/1/30	3.168	4.443	0.788	12.520
2009/4/30	2.813	4.219	1.056	12.955

$$v_t = \left(\frac{1}{N_t} \sum_{i=1}^{N_t} (P_{it}(0) - E(P_{it}(0)))^2 \right)^{\frac{1}{2}}.$$

Conclusions

- We propose a method to estimate recovery rates and default probabilities of investment corporation bonds using corporate bond pricing model proposed by Kariya and Tsuda (1994) and Tsuda (2003).

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