

# TEST STATISTICS, OPTIMAL ORBITS, AND CRIMES:

## Theory and Empirical Evidence

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**ABSTRACT:** **The objective** is to optimize a region; general equilibrium policy is the unique smooth orbit of global and local consistency, transforming the correlation, and controlling and altering the direction of potential natural forces. **Method** is that dynamic quadratic regression (or non-deterministic polynomials) yields general equilibrium and spectrum. **Outcomes** are that the data accuracy may follow the Gaussian distribution,  $x(t) \sim N(0, \sigma_x^2)$ , or the non-Gaussian distribution,  $x(t) \sim N(x^*, 0)$  at time  $t$ . Equilibrium  $x^*$  is the turning point, the minimum and maximum in policy and welfare effect size, and is located for all time or ages. Conflicts and errors vanish if the convergence probability  $p \rightarrow 1$  is the criterion of acceptability, controllability, and incentive-compatibility for unique budget coverage, cooperation and competition in interdiscipline problems.

The target is that equilibrium is the smooth orbit of flows for long-time existence in the interaction of  $n$ -codimensional spheres. The curse of dimensionality ( $D$ ) is the small sample size with a large number of conflicts or unknown parameters,  $n < t$ . The equilibrium solution is the prediction and a unique, unbiased, consistent, and efficient test statistic, and is a boundary and fractional optimization, classifying normality and irrationality.

**Keywords:** Convergence probability; General equilibrium in continuous space and time; non-deterministic polynomial ( $NP \neq P$ ) problems on conflicting theories.

**AMS subject classification.** 49K40, 35B37.

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## 1.INTRODUCTION

The price is gains from the positive excess demand for commodities, human trafficking, or crimes. The global penalty prices are transnational penalties for transforming the excess demand into negative excess demand, preventing human crimes. Thus, crime penalties are a time-invariant source of funding against crimes. Huck, et al.(2011) considered the multiperiod deferred compensation and effort; when the firms cannot commit to its offer, the end result is a low-wage, low effort equilibrium; social interference is required in the face of distribution conflicts; the subjects' self-belief affects the higher productivity and the preference for variable payments of tournament. However, Debreu'(1984) general equilibrium is static, uses Shizuo Kakutani's fixed point, and proves the local existence of equilibrium prices at zero excess demand. Few works have transformed the prices or correlations into general equilibrium for the smooth orbits and fractional optimization .

Here, in this essay, we compute non-zero equilibrium for empirical evidence. In n-commodity or n-codimensional(nD) spheres,  $v(t)$  denotes crimes, errors, or perturbations. We add the time t, and show that equilibrium, zero or non-zero, solves the interdisciplinary problem for n-bodies. The functions  $F(.)$  of production and utility are convex and concave curves and intersect at the equilibrium. The objective is:

$$(1.1) \text{Maxmin } H_t(Y, x, r, \lambda) = \sum_{t=1}^n (F_t(Y, x, r, \lambda) - Y^*)$$

for  $0 \leq x(t/n)=r(t/n)=t/n \leq 1$  as  $n \rightarrow \infty$ ,

$$(1.2) \quad \lambda_0 = (1.2) \quad \lambda(0); \quad Y_0 = Y(0); \quad x_0 = x(0); \quad r_0 = r(0)$$

where (1.1) is a utility and production function; consumers maximize the utility preference of output; producers minimize the cost of inputs  $(Y, x, r)$ . (1.2) is the initial values or endowments.  $(H, F, p)$  are thrice differentiable functions.  $Y^*$  is the equilibrium volume or potential endowments.  $\lambda = \theta_t$  is the Lagrangian multiplier, or opportunity prices or costs.  $x = d \log Y / dt$  is the growth rate or responses.  $r$  is a control policy or stimulus.

THEOREM 1: The stability implies that equilibrium  $x^* = x(t)$  is a stable global attractor, zero or non-zero, is a common divisor,  $x^* = \sum_{t=1}^n x(t)p(x(t))$ , and a unique, unbiased and consistent test statistic or boundary for classification or identification.  $p(x)$  is the convergence probability. Equilibrium is orthogonal to the correlations  $\theta_t$  and resolves the multicollinearity and non-independence of errors.

Below or beyond the equilibrium, a commodity is an inferior or superior good; the equilibrium price is a positive compensation or a negative penalty for a commodity, a service, or a crime,.

The Walras' law is

$$(1.3) \quad \lambda_t F_t(\lambda_t, Y) = v(t) \quad \text{if } v(t) = d \log Y(t) / dt - x^* = x(t) - x^*$$

where the estimate  $Y^*$  of equilibrium quantity  $Y$  can be dependent or independent of the equilibrium prices  $F(\lambda_t) = 0$ .  $Y(t) - Y^* > 0$  is the excess supply; and *vice versa*. In equilibrium price,  $Y(t) = Y = Y^*$ :

$$(1.4) \quad F_t(\lambda_t, Y) = 0, \quad \text{for } v(t) = 0, \quad \lambda_t > 0 \text{ if } Y(t) < Y^* = Y; \text{ and } \lambda_t > 0 \text{ if } Y(t) < Y^* = Y$$

where in non-equilibrium, the price or the elasticity of demand,  $\lambda_t = \theta_t > (\text{or } <) 0$  is sign-changing; the utility preferences can be convex or concave:

$$(1.5) \quad [x(t) \leq (\text{or } \geq) x^*] \Leftrightarrow [F(x(t)) \leq (\text{or } \geq) F(x^*)] \quad \text{if } x(t) = t \leq (\text{or } \geq) x^* = n$$

where the notation “ $\Leftrightarrow$ ” implies the direction of utility preference “ $\geq$ ”. The gain or profit solution is determined by the endogenous variables or equilibrium, including output and inputs.

The frequency of flows of output or crimes depends on organizations and environments. In prisoners’ dilemma, the best strategy is equilibrium or zero crimes. Prisoners react like a mouse in a cages or a bipolar patient in a confinement of idea choices, ignore alternatives, and have only two choices between “yes or no”, (0 or 1), or admitting and no admitting the crimes, or cooperation and competition, or the white-light space and dark space. In case of human trafficking, if the monopoly of crimes is penalized by severity or by the count of the number of heads-trafficking, then the elasticity of crime demand could be negative or inelastic:

$$(1.6) \quad \log Y(t) = \theta_i t + v(t), \text{ for } d \log Y(t) / dt = \theta_i < 0$$

where  $Y(t)$  is flows of responses, output or funding. As the price of crime penalties increases with crime compensations, the frequency of flows increases:

$$(1.7) \quad \log Y(t) = \theta_i x(t) + v(t), \text{ for } d \log Y(t) / dx(t) = \theta_i > 0$$

where under monopolistic and imperfect competition, the elasticity of demand is positive  $0 < \theta_i x(t) < 1$ ; under the perfect competition, the elasticity tends to be infinity.

**THEOREM 2:** Regularity implies that equilibrium is the regulatory standard of the maximum likelihood function  $\log F_i(Y) = 1$ , is invertible or reciprocal  $(1/\log Y(t)) = 1$  between responses  $x(t)$ , and stimulus  $r(t)$ , and is efficiency indicator of resource allocation against crimes and monopoly frequency

$$(1.8) \quad p(Y) = \exp(\theta_i dH(Y(t))/dt) / dt = \exp(\theta_i (\log Y(t) / \log Y^*)) \leq 1$$

for  $\theta_i < 0$  if  $\log Y(t) / \log Y^* > 1$ ; and for  $\theta_i > 0$  if  $\log Y(t) / \log Y^* < (t/n) = 1$ ;

where  $Y(t)=F(Y;x,r;t)$ .

Equilibrium minimizes the variance or uncertainty, and satisfies the first, second, and higher order derivative of optimization:

$$(1.9) \quad p(x)=\exp(\theta_t dH(Y(t))/dt)/dt \\ =\exp(\theta_t (\log Y(t) -\log Y^*)^2) \leq 1 \quad \text{for } 0<t\leq n>2 \\ =\exp(\theta_t \sigma_{\log Y}^2 +v(t))$$

where  $p(Y)$  is the convergence probability, acceptability and controllability. equilibrium price, or penalty cost  $\lambda_t=\theta_t$ ; in equilibrium, the errors or crimes  $v(t)$  tends to vanish.

Theorem 3: :Equilibrium is a zero or non-zero smooth orbit of flows. Below or beyond the equilibrium, the direction of flows and variations is sign-changing.

$$(1.0a) \quad 0 \leq x(t/n)=r(t/n)=t/n \leq 1, \quad \text{if } \theta_t > 0, \text{ and } x(t) < x^*(n)=n, \text{ as } n \rightarrow \infty$$

$$(1.0b) \quad 0 \geq \theta_t x(t/n)=\theta_t r(t/n)=\theta_t t/n \geq -1, \quad \text{if } \theta_t < 0 \text{ and } x(t) > x^*(n) \text{ as } n \rightarrow \infty$$

where the target equilibrium  $x^*=x(t)$  is the attractor and an intersection point, pulling backwards, pushing forwards the group's elements  $\{x(t)\}$ .

In economy, below or beyond the equilibrium, n-goods, such as the government debt or paper currency become risky loans, or crime assets. As the quantity of debts or paper currency increases or decreases, the prices fall; while the price of alternatives, such as gold price rises, conserving the money-functional value.

The growth objective is to maximize the Hamilton-Jacobi-Bellman equation  $H$ , and minimize the errors or deviations.

$$(2.1) \quad H(t)=F_t(x;Y,r)+\lambda_t(x_t-x^*)^2$$

where  $(H, F, p)$  are thrice differential functions. Equilibrium is consistent with

respect to time and estimated by the dynamic quadratic regression, minimizing the mean square errors(MSE),  $v^2$ :

$$(2.2) \quad dH(x(t))/dt = \text{minimize} \left( dx(t)/dt - \theta_t \frac{\partial^2 F_t(x; \theta)}{\partial x^2} \right)^2$$

$$\text{if } dx(t)/dt = \theta_t \frac{\partial^2 F_t(x; \theta)}{\partial x^2} + v(t) \quad \text{for } n > 2$$

In astronomy, equilibrium is the smooth orbits for long-time existence of n-bodies, such as sun, moon, and earth. Below or beyond the equilibrium, equilibrium is a spectrum boundary on an equator or north and south poles, the correlation is sign-changing, conserving the potential energy:

$$(2.3) \quad -\pi \leq -1/2) \quad \pi \leq 0 \leq \theta_t \leq (\pi/2) \leq \pi, \quad \text{if } \theta_t \geq 0, \quad \text{for } x(t) = t \leq x^* = n$$

$$(2.4) \quad -\theta_t \pi \geq -\theta_t 1/2) \quad \pi \geq 0 \geq \theta_t \geq (\theta_t \pi/2) \geq \theta_t \pi$$

$$\text{if } \theta_t < 0, \quad \text{for } x(t) = t > x^* = n$$

Phase 1: In phases of evolution waves, every body self revolves, and solves for equilibrium  $x^*$ :

$$(2.5) \quad p(x) = \exp(dx/dt) = \exp(\theta_2 (x_t - x^*)^2 + v(t)) \quad \text{for } 0 \leq t \leq n \text{ and as } n \rightarrow \infty.$$

$$\text{if } x(t) = \theta_t x(t-1) + v(t), \quad \text{if } \theta_t \geq 0, \quad \text{for } x(t) = t \leq x^* = n$$

$$\text{if } x(t) = \theta_t x(t-1) + v(t), \quad \text{if } \theta_t < 0, \quad \text{for } x(t) = t > x^* = n$$

where  $p(x) = \mathfrak{R}^2$  is the convergence probability, and  $\mathfrak{R}^2$  is the coefficient of determination in regression.  $v(t)$  is errors.  $\theta_t$  is a correlation or angle,  $x = r \cos^2 \theta_t$ .

In two dimensional(2D) histogram or distribution domain, the horizontal axis is the time  $t$ ; the vertical axis is the frequency  $x = x(t/n) = t/n$ . Equilibrium is a latent or unobservable variable, reflecting the direct and indirect effect of the correlation  $\theta_t$  and variables  $\{x(t)\}$ .

Phase 2: Two bodies mutually revolve and form an equator in hyperbolic or parabolic equation:

$$(2.6) \quad p(x) = \exp(dx/dt) = \theta_2 (x_t - x^*)^2 + \theta_4 (r_t - r^*)^2 + v(t)$$

where the global attractor is the upper and lower bound and the confidence interval:

$$(2.7) \quad \theta_t \sigma - x(t) < x^* < x(t) + \theta_t \sigma; \text{ and } \theta_t \sigma - r(t) < r^* < r(t) + \theta_t \sigma;$$

Phase 3: Three bodies form a public revolution as if output growth, interest rates, and money supply growth interact at the exchange rate.

$$(2.8) \quad p(x) = \exp(dx/dt) = \theta_2 (x_t - x^*)^2 + \theta_4 (r_t - r^*)^2 + \theta_6 (\log Y_t - \log Y^*)^2 + v_t$$

where the confidence interval for the system is:

$$(2.9) \quad Y(0) \exp(\theta_t \sigma t) - Y(t) < Y^* < Y(t) + Y(0) \exp(\theta_t \sigma t)$$

where the equilibrium,  $x(t) = x^*$ , is the gravitation center and fills the gaps of elements. Below or beyond the equilibrium, the correlation  $\theta_t \sigma > (\text{or} <) 0$  is sign-changing:  $\sigma$ ; and  $\theta_t \sigma - r(t) < r^* < r(t) + \theta_t \sigma$ . In nonequilibrium, the responses and stimulus show oscillations:

$$(2.10) \quad Y(t) = Y(0) \exp(xt) \exp(-rt)$$

$$Y(t) \geq Y(0) \quad \text{if } x(t) = t \geq x^* = r = n$$

$$Y(t) < Y(0) \quad \text{if } x(t) = t < x^* = r = n$$

Theorem 4: *The equilibrium prime and dual  $\lambda_t Y^*$  is the exercise or strike price.*

Equilibrium is the upper and lower bound; beyond or below the equilibrium, the equilibrium price  $\lambda_t = x(t) = t$  is the predictive time to fall or to rise. The option is exercised to contract or expand, or sell or buy the underlying assets at the contract price or exercise price  $\lambda_t Y^*$ ; the exercise time is  $0 \leq x(t) = t \leq n$  before or on the maturity date  $n$ .

Suppose  $x$  is the risk premium or gain:

$$(2.1) \quad x = \text{maximize}(Y(t) \exp(-rt) - Y^*, 0) \quad \text{if } Y^* > Y(t) + \theta_t \sigma_Y$$

$$(2.2) \quad x = \text{minimize}(Y^* - Y(t) \exp(-rt), 0) \quad \text{if } Y^* < Y(t) + \theta_t \sigma_Y$$

where exercise prices  $\lambda_t Y_t = \lambda_t Y^*$  is the prediction and expected value, trimming out uncertain risk, and stabilizing the portfolio values.

THOREM 5: Below or beyond the equilibrium ratio  $(G-T)/Y$ , the government debts,  $D$ , are safe assets, or risky loans, when the debts or government deficits  $(D/Y) = (G-T)/Y$  are supported by the excess of output growth over interest rates.

Figures 1,2,3 and 4 show that equilibrium is the upper and lower bound. For empirical evidence on the following examples, see Hsieh(2002).

$$(3.1) \quad Y(t) = F_t(x, r) \quad \text{if } A = K + D \quad (\text{Asset} = \text{Capital} + \text{Debt})$$

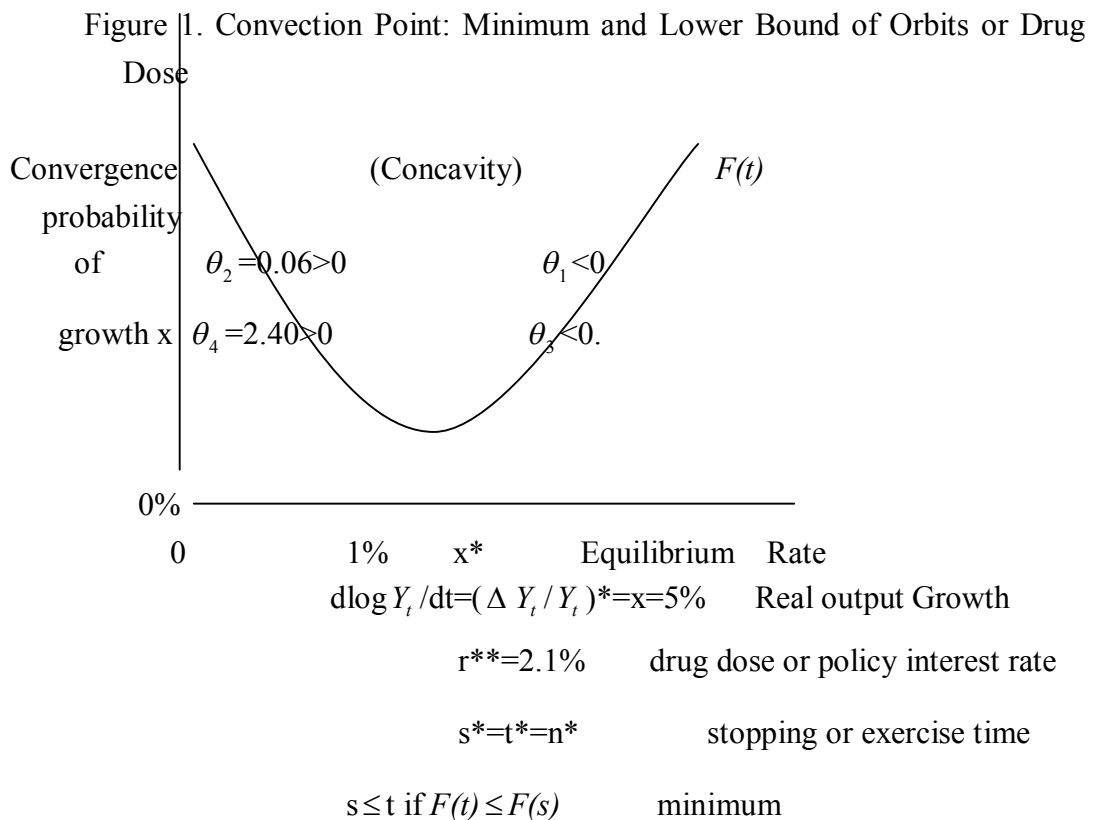


Figure 2. Maximum And The Upper Bound Equilibrium ( $Y^*$ ,  $x^*$ ,  $t^*$ ) of orbits, or drug effects or welfare improvement



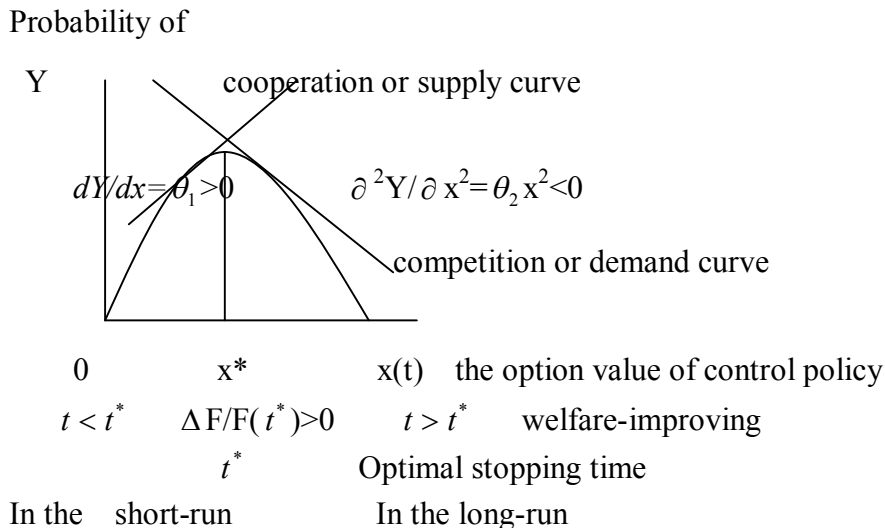


Figure 3: Equator (or Exchange Rate)

:Equilibrium is the smooth orbit of flows for three n bodies.

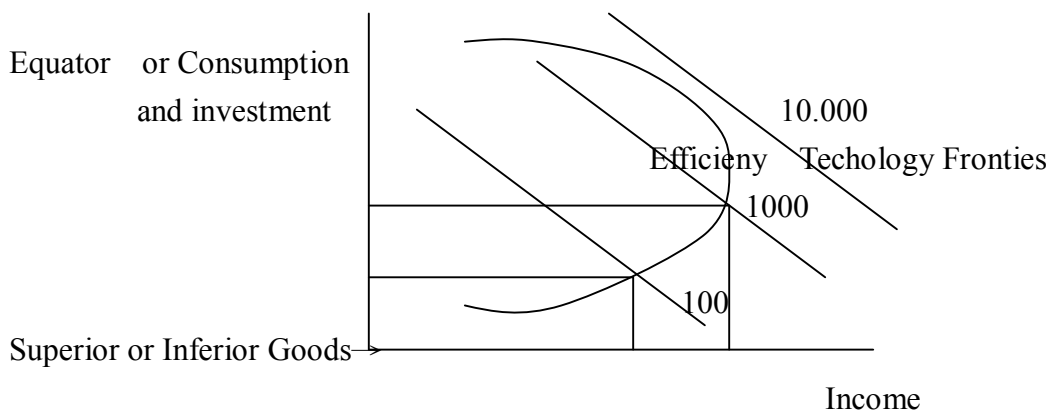
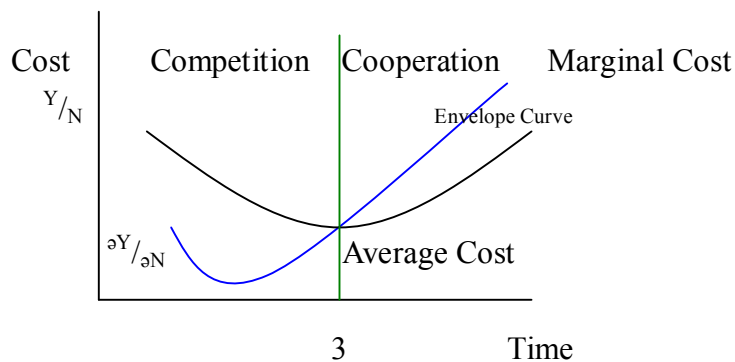


Figure 4 : Envelope curve Implies a curve of long-run existence



In equilibrium, Average and marginal cost curves intersect.

Average cost,  $Y/N$ , = Average output.

## VI. AGGREGATION AND INTERVENTION

PROBLEM 3. On unique budget coverage: General equilibrium is the maximum effective policy and the minimum in drug doses. The prediction  $Y_{t+n}$  is recursive and not constant at n-steps ahead and has a correlation or direction  $\theta_t$  at time t and sample or population size n:

$$(4.1) \quad F(\log Y_{t+n}) \approx F(\log Y_t) + [\lambda] \frac{dF(\log Y_t)}{dt}$$

where (4.3a) is the ill-posed, linearized problem of invertibility (Jin, et al. 2009, p. 1997). The first-order direction  $[\lambda] = \theta_t$  is the correlation and the Jacobian compact operator and the Robin constant coefficient for multiple first-order derivatives.

PROBLEM 4: On the micro and macro consistency. The risk premium is

$$(4.2) \quad Y(t)/Y(0) = \exp(x(t) - r^*)t = \frac{D_t}{D_t - T_t} \quad (\text{Agoglia, et al. 2011})$$

$$= \frac{(1 - (K_t / A_t))}{(1 - (K_t / A_t)) - (T_t / A_t)}$$

where  $r^*$  is the equilibrium cost of capital; the anti-Keynesian theory would suggest the austerity policy, reduce the government spending and enhance the tax revenues, and balance the budget deficit,  $T - G < 0$  towards  $> 0$ . The unique budget coverage is balanced.

Here, we show that equilibrium is a unique, unbiased, and consistent test, and is a precision accounting standard with order of errors  $O(\theta_t x^{-n})$ . Below or beyond the equilibrium, debts  $D$  or income  $Y$  could increase through exercising the options at the strike or equilibrium price  $\lambda_t$ , such as selling the underlying assets  $A$ , losing or earning the risk premium,  $x - r > (or <) 0$ , or institutional and civil service reforms.

THEOREM 6: The duality  $\lambda_t = 1 = q_t = P(t)/P^*$  is equilibrium price and intervention controller;  $x < r = \lambda_t < (1/x)$ ; general equilibrium policy is a balance center, satisfying

the identity, and minimizing the mean squared errors. Below or beyond the equilibrium, the risk preference and utility changes.

An identity is equilibrium:

$$(4.3) \quad Y_t = x_t + r_t K + w_t N_t = C_t + (K_t - K_{t-1}) + G_t + X_t$$

$$= C_t + \text{Saving} + T_t = A_t K_t^{\theta_1} N_t^{\theta_2} e^{v_t}$$

where in (4.4),  $Y$  is output, income, expenditure, and production;  $x$  is output growth, the surplus, or Solow's residuals with delay equilibrium distribution;  $rK$  is the interest rate of capital and rental of land; and  $wN$  is the wage income of labor numbers  $N$ .  $C$  is consumption;  $A$  is technology;  $K$  is capital;  $v=dx/dt$  is residual errors or perturbations;  $G$  is the government spending;  $X$  is net exports or net imports;  $T$  is tax revenues;  $G-T>0$  is debt or budget deficit;

Output falls below the expected costs,  $\log Y < x + r + w + v$ , if output is smaller than the surplus, the interest rate, wage, and rental ratio or depreciation. Crises of debts default are avoided if the cumulated debts/income ratios  $B/Y$  are stabilized, if the cost of rental and interest rates  $r \leq x$  is covered by output growth; while the budget deficit and money growth are in balance. In  $n \geq 3$  three phases, output growth is Solow's residual or surplus  $x(n)$ ; and net exports  $X(n)$  or inventories  $X$  are treated as assets in general equilibrium in continuous space and time. The equilibrium exchange rate is the first and second or higher -order consistent derivative, and supported by the forwards and backwards international interest discount rates and the purchasing power parity.

The primal  $(Y, x, r)$  and the dual  $q = \lambda_t$  interact:

$$(4.4) \quad (P(t)/P(t^*)) = q_t = (1/r_t)/(1/x^*) = 1,$$

$$(4.5) \quad dq/dt = dF_t(q, r)/dt = \theta_2 (q_t - q^*)^2 + \theta_4 (r_t - r^*)^2 + \theta_6 (x_t - x^*)^2 + v_t$$

$$\text{if } r_t = r^*, \quad q_t = q^* \text{ for } t > 0$$

where dynamic Tobin's  $q$  of market present value requires the marginal cost and

benefit  $r=x=t/n$  to be equal:

$$(4.6) \quad q_t = P(t)/P(t^*) = YP(t)/YP(0)(P(t^*)) = ((1+(x(t)-r(t))) \geq 1 \quad \text{if } x(t)=t \geq x^* = r, \\ = 1/(1+(x(t)-r(t))) \leq 1 \quad \text{if } x(t)=t \leq x^* = r$$

where equilibrium exchange rate  $q^*$  is the ratio of the market value and the initial book value, where  $P(t)$  is the domestic price of say, the bond price;  $P(x(t))=1/r$ ; is the foreign price in the long run, and  $YP(r(t^*))=YP(0)\exp(-rt)$  in the short run.

Equilibrium policy is  $(q_t=q^*, r_t=r^*=r)$  if  $v=0$  for  $t>0$ , which is consistent with the potential ratio  $r^* \geq (or \leq)x^*$ , and which is supported by the currency -substitutable regulatory standard, and the latent invisible value of, say, intellectual property rights.

$$(4.7a) \quad ((d \log Y/dt - \theta_1 x - \theta_2 x^2 - \theta_3 r - \theta_4 r^2) / \frac{T-G}{Y}) = \theta_5 \quad (\text{mod } \frac{T-G}{Y})$$

$$(4.7b) \quad ((d \log Y/dt - \theta_1 x - \theta_2 x^2 - \theta_3 r - \theta_4 r^2 - \theta_5 \frac{T-G}{Y}) / (\frac{T-G}{Y})^2) = \theta_6 \\ (\text{mod } (\frac{T-G}{Y})^2)$$

where  $x(0)=1$ ,  $\theta_0$  and  $v(t)$  will be treated; deviations vanish in equilibrium.

Equilibrium is governance and a regulator;  $T$  is tax revenues;  $G$  is the government spending.  $(T)-G) \approx B$ , denoting debt or budget deficit.

The wave-evolution equation is solved at equilibrium:

$$(4.8) \quad d \log Y/dt = \theta_2 (x_t - x^*)^2 + \theta_4 (r_t - r^*)^2 + \theta_6 (\frac{T_t - G_t}{Y_t} - \frac{T^* - G^*}{Y^*})^2 + v(t)$$

where output  $Y$  oscillates under perturbations  $v$ .

$$(4.5b) \quad dq/dt = dF_t(q, r)/dt = \theta_2 (q_t - q^*)^2 + \theta_4 (r_t - r^*)^2 + \theta_6 (x_t - x^*)^2 + v_t$$

$$\text{if } r_t = r^*, \quad q_t = q^* \text{ for } t > 0$$

where dynamic Tobin's  $q$  of market present value requires the marginal cost and benefit  $r=x=t/n$  to be equal:

## V. EMPIRICAL EVIDENCE

### EXAMPLE 1: Monetary Policy

Suppose the monetary and interest rate policy is incentive-compatible with the maximum equilibrium growth and inflation tax in Taiwan. The data used cover the period 1983:1 - 2001:4, as published by the government in Taiwan. We estimate that the maximum output growth is 9% and the optimal real interest rate is 2.5% with a standard deviation 2%. The average real interest rate is  $r=1.9\%$  with a standard deviation 7.8%. The quarterly data are more able to reflect the dynamic cycles.

Suppose  $x = d \log Y_t / dt$  is output growth;  $r$  is the real interest rate.  $M$  is the money supply ( $M1$ ).

Step 1: Start with the initial values of the sample period. Define the objective function:

$$(5.1) \quad x^* = \arg \sup_{x^* \in \Omega} F(x, r, d \log M / dt, t) \quad 0 < t \leq n$$

where  $\Omega$  denotes a well-posed organization in the continuous, compact, and convex and concave domain, where the equilibrium exists and unique.

Step 2: Compute the dynamic quadratic regression

$$(5.2) \quad \begin{aligned} \Delta x(t) = & -3.48 + 0.09x(t-1) + 0.05x^2(t-1) + 0.73r(t-1) - 0.13r^2(t-1) \\ & (1.27) (0.19) \quad (0.01) \quad (0.56) \quad (0.067) \\ & (-2.74) (0.46) \quad (3.70) \quad (1.30) \quad (-1.93) \\ & + 0.07(d \log M / dt) - 0.005(d \log M / dt)^2 \\ & (0.06) \quad (0.001) \\ & (1.12) \quad (-3.70) \\ & + 0.50 \Delta x(t-1) - 0.20 \Delta x(t-2) \\ & (0.11) \quad (0.08) \\ & (4.42) \quad (-2.31) \end{aligned}$$

$$R^2 = 0.78 \quad \bar{R} = 0.75 \quad n = 76 \quad D.W. = 1.61 \quad 1^{\text{st}} \text{ order autocorrelation} = 0.18$$

where the upper values in the parentheses are standard errors; the lower values are Student  $t_n$  statistic. For seasonal data,  $\Delta x(t) = x(t) - x(t-4)$ ; and  $x = d \log Y / dt \approx (Y_t - Y_{t-4}) / Y_{t-4}$  is the annualized growth rate of four-quarterly national income  $Y$ .

Step 3: Compute the equilibrium.

From (5.2), the output growth is  $x^* = \theta_1 = 0.09 = 9\% > 0$ . For  $\theta_1, \theta_2 > 0$ ,  $x^*$  is the instable solution. The maximum real interest rate is  $r^{**} = -0.73/(2)(-0.13) \approx 2.5\%$ ; the maximum growth rate of money supply M1 is  $d\log M^*/dt = -0.07/2(0.005) = 7\%$ .

Step 4: Reduce the heteroschedasticity of errors, and reestimate the delay time period and the convergence probability:

$$\begin{aligned}
 (5.3) \quad d\log Y/dt &\approx \frac{x(t) - x(t-4)}{x(t-4)} \\
 &= 13.88 - 0.49(x(t-1) - 9\%)^2 + 0.51(x(t-1) - 9\%)(r(t-1) - 2.5\%)^2 \\
 &\quad (7.58) \quad (0.23) \qquad (0.18) \\
 &\quad (1.83) \quad (-2.10) \qquad (2.75) \\
 &\quad + 0.86 \frac{x(t-1) - x(t-5)}{x(t-5)} - 0.37 \frac{x(t-2) - x(t-6)}{x(t-6)} \\
 &\quad (0.108) \qquad (0.11) \\
 &\quad (7.93) \qquad (-3.32)
 \end{aligned}$$

$$R^2 = 0.63 \quad \bar{R} = 0.61 \quad n = 76 \quad D.W. = 2.09 \quad 1^{\text{st}} \text{ order autocorrelation} = -0.04$$

where the convergence probability is  $p(x) = R^2 = 0.63$ . Output growth,  $0 \leq x^* \leq 9\%$ , has an upper bound of the equilibrium growth,  $x^* = 9\%$ . The delay response timing is  $\Delta x(t) = -0.37 \Delta x(t - t^*)^2$ . The equilibrium effect of policy takes a delay time,  $t^* = (0.86)/(2)(-0.37) = 1.2$  years. Here, the delay impact time is about a one-year horizon.

#### EXAMPLE 2: Real Exchange Rate

The equilibrium policy is a unique, unbiased forward and backward, and ex post consistent solution under the floating foreign exchange rate regime. Suppose  $q$  is the real exchange rate on the pound/dollar, which is adjusted by the value-added price deflators. The quarterly data of the real exchange rate over 1978.1 to 1996.4 are used, as reported by *International Financial Statistics*. Equilibrium is the unbiased prediction of the pound/dollar exchange rate, and is predicted by the following dynamic quadratic regression.

$$(5.4) \quad q = \arg \min_{q \in \Omega} (\max_{0 < t \leq n} F(x(t))) = \arg \max_{q \in \Omega} (\min_{< t \leq n} F(x(t)))$$

where  $t$  denotes time or quarters. Our unique equilibrium of the real exchange rate,  $q^*=1$ , is unbiased as follows:

$$(5.5) \quad \Delta q = q(t) - q(t-1) = \theta_0 + (\theta_1 - 1)q(t-1) + \theta_2 q^2(t-1) \\ = -0.17 + 0.46q(t-1) - 0.23q^2(t-1) + v(t)$$

$$(0.8) \quad (3.38) \quad (-1.2) \\ = -0.23(q(t-1) - (0.46)/2(0.23))^2 + v(t)$$

$$\bar{R}^2 = 0.8; \quad \text{D.W.} = 1.71$$

where  $v$  is residual errors. Student  $t_n$  statistics are reported in the underlying parentheses, but are biased due to endogeneity and autocorrelation.

EXAMPLE 3: *On unemployment and real interest rates:*

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Let  $u$  be the unemployment rate.  $r$  is the real interest rate,  $r = i + \Delta P(t)/P(t)$ , which is the nominal interest rate  $i(t)$ , plus the price inflation rate.  $0 < t \leq n$  is time or quarterly.  $n$  is the sample size and a positive integer.  $\Delta u = u(t) - u(t-1)$ . The ordinary least squares regression is estimated as follows:

Using the Taiwanese quarterly data over the period 1977.4 through 2001.4, the trade-off between unemployment and inflation is more significant than between unemployment and the real interest rate. The data are published by the statistical and auditing bureau, the government in Taiwan.

In other words, the equilibrium unemployment  $u^*=2\%$  is attained through the real interest rate  $r^*=2\%$ . Similarly, the equilibrium economic growth is around  $x=5\%$ .

$$(5.5) \quad \Delta u = -0.04 + 0.006(u(t-1) - 2\%)^2 + 0.003(r(t-1) - 2\%)^2 + 0.006(u(t-1) - 2\%)(r(t-1) - 2\%)^2 \\ (-0.97) \quad (3.19) \quad (1.47) \quad (2.07)$$

$$\mathfrak{R}^2 = 0.11 \quad \bar{\mathfrak{R}}^2 = 0.11; \text{ .D.W. } = 2.2; n=92; 1^{\text{st}} \text{ order autocorrelation } \rho = -0.139$$

where the values in the underlying parentheses are Student t statistic. n is the sample size.

The unemployment rates are determined by nature and nurture; we find the coexistence of the positive and the negative impacts of government spending around a turning point. According to neo-classical theory, the unemployment rate is assumed to increase, as the government spending crowds out private investment and consumption. According to the Keynesian economics, when government spending increases the effective demand, the unemployment rate decreases. In Egypt in 2010-2011, the social-science graduate students are excessively supplied and have 20% unemployment rates.

REMARK: In financial history, equilibrium interest rate is fair. In a period of the gold standard, 1863-1887, the Bank of Portugal was a central bank; and the government normally opposed the increases of the nominal discount rate above 5 per cent. This equilibrium corresponds to the liquidity ratio of official reserves to sight obligations( bank notes and deposits (Reis, 2007, page 721).

*Example 4:* The balanced budget rule, no crowding-out effect, and economic growth in Taiwan. The maximum equilibrium of output growth is 5% , the maximum government spending is 25%, and the maximum flat income tax rate is 23%. When the government spending exceeds the optimum share  $(G/Y)^{**} > 25\%$ , and the tax revenues exceed  $(T/Y) > 23\%$ , the expansionary fiscal policy crowds out investment and consumption, and reduces output growth.

$$(5.6a) \quad dx/dt = -0.08 + 1.27x(t) - 0.57x^2(t) + 0.63(G(t)/Y(t)) - 1.36(G/Y)^2 \\
\begin{matrix} (-4.05) & (6.01) & (-4.90) & (2.45) \\ +0.68(T(t)/Y(t)) - 1.77(T(t)/Y(t))^2 + v_4(t) \\ (1.89) & & (-1.41) & \end{matrix}$$



$$\bar{R}^2=0.77; \quad D.W.=1.87$$

$$(5.6b) \quad \Delta x = -0.57 (x(t) - 0.05)^2 - 1.36 ((G(t)/Y(t) - 0.25)^2 - 1.77 (T(t)/Y(t) - 0.23)^2 + v_4(t)$$

where  $x = d \ln Y / dt$  is output growth.  $Y$  is output (GDP);  $G$  is government spending;  $T$  is tax revenues.  $\theta_2 = -0.57 < 0$ ,  $\theta_4 = -1.36 < 0$  and  $\theta_6 = -1.77 < 0$  imply that output growth is at the maximum 5% and starts to decrease if the government spending ( $G/Y$ ) and the income tax rate ( $T/Y$ ) deviate from the optimum, respectively. The stabilization policy of real interest rate,  $0.8\% < r^* < 2\%$ , is actually achievable by the maximum efficient share  $(G/Y)^{**} = 25\%$  of government spending in GDP and the maximally efficient income tax rates,  $(T/Y)^{**} = 23\%$ .

## V. Conclusion and Insight

Non-zero general equilibrium is a precision measurement of the fittest, like eye-cells, using a camera structure, projecting the different angles or correlation. Eye-cells adjust the focus from two angles  $\theta_i$ . The time-consistent distance  $t$  always exists and is a common divisor, locating the unique diagnosis of flow speeds and orbits in space-like and light-like time.

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